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ABSTRACT

This working paper presents specifications for the test items to be used in the second mathematics study to be conducted by the International Association for the Evaluation of Educational Achievement (IEA). A content-by-behaviors grid is presented for two population levels, with specifics for each dimension outlines and examples of test items provided. (MS)

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Wellington, January 1977.

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SECOND IEA MATHEMATICS STUDY

SUGGESTED TABLES OF SPECIFICATIONS FOR THE IEA MATHEMATICS TESTS

WORKING PAPER I

Prepared by the IEA International Mathematics Committee

NOTE: Comments, queries, suggestions and criticisms are requested from National Centers and National Mathematics Committees together with the information requested in the questionnaire attached (see page 96.)

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I Introduction

The purpose of this IEA working paper is to present a reference document on the specification of mathematics test items to be used in the second mathematics study. A content X behaviors grid is described as a table of specifications for mathematics test items at each of the population levels (Population A and Population B. See Bulletin No 1, p 18 for definitions.)

The material presented in this working paper was produced by groups at the Urbana and St Andrews conferences, and adapted from the chapter by Wilson in the Handbook On Formative and Summative Evaluation of Student Learning.*

The document is largely in outline form because we believe this form is most useful at the moment. Furthermore, we anticipate it will be refined as we receive responses from members of the IEA International Mathematics Committee, National centers, or other interested mathematics educators.

*Wilson, J W, Evaluation of learning in secondary school mathematics. In B S Bloom, J T Hastings, and G F Madaus, Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw-Hill Book Company, 1971.

II Tables of Specification

Our task is to find an organizational strategy for examining mathematics achievement and mathematics curricula. One strategy widely used is to develop a content X behaviors grid where on one hand mathematics achievement, objectives, or curricula are described in terms of a detailed content outline. On the other hand achievement on a particular content category can be described in terms of the level of cognitive complexity expected of students' behavior. In this section the grids for each of the populations are presented. The following sections provide discussion and illustration of the grid categories.

Similar strategies of content X behaviors description of mathematics achievement have been widely used over the past 20 years. The 1964 IEA mathematics survey utilized such a device. The previously mentioned chapter by Wilson provides a review of the uses of many such content X behaviors descriptions and provides ties to more general taxonomies of educational objectives.

There is potential for confusion of the terms "application" and "analysis" as used to describe mathematics content and "application behaviors" and "analysis behaviors" as used to describe levels of behavior or the cognitive complexity of a task. However, these terms have a rather long history in the literature and are adequately described in later sections and by the context in which they are used.

IEA MATHEMATICS STUDY

Table of Specifications

Population A

	Computation Behavior	Comprehension Behavior	Application Behavior	Analysis Behavior
000 Arithmetic				
100 Algebra				
200 Geometry				
300 Probability & Statistics				
400 Measurement				

IEA MATHEMATICS STUDY

Table of Specifications

Population B

	Computation Behavior	Comprehension Behavior	Application Behavior	Analysis Behavior
1. Sets, Relations & Functions				
2. Number Systems				
3. Algebra				
4. Geometry				
5. Analysis				
6. Probability & Statistics				
7. Finite Mathematics				
8. Computer Science				
9. Logic				

III Mathematics Content

The mathematics appropriate for Population A is described by the broad content areas of arithmetic, algebra, geometry, probability and statistics, and measurement. For most countries, topics under the arithmetic heading will constitute much of the appropriate mathematics experience. The outline that follows, however, was written to be comprehensive and provide for inclusion of topics that may be included in any mathematics curriculum of an IEA country.

The mathematics content description of Population B is a much more encompassing outline of mathematics. This is because of two things. First, the pre-university year has more depth and variety in its mathematics content. Second, across schools and countries there is tremendous variety of mathematics in the pre-university year and the outline here attempts to be inclusive.

An elaboration on the outline for Population A is given in Appendix I. This elaboration contains many example items to help communicate the range of content within each category.

Outline: Population A

Content Dimension for Population A

000 Arithmetic

001 Natural numbers and whole numbers

001.1 Place value

001.2 Operations (function, algorithms)

001.3 Representation on number line

001.4 Properties of operations

001.5 Ordering whole numbers

001.6 Estimating products, sums, quotients, differences

002 Common fractions

002.1 Fractional parts

002.2 Order, equivalent fractions

002.3 Representation on a number line

002.4 Simplification of fractions

002.5 Mixed numerals

002.6 Density

002.7 Operations (addition, subtraction, multiplication, division)

002.8 Approximation to simpler fractions

002.9 Estimation

003 Decimal fractions

003.1 Common fractions with denominators a power of ten

003.2 Place value

003.3 Order of decimal fractions

003.4 Representation on number line

003.5 Representation of rational numbers by decimal fractions

003.6 Estimation and approximation

003.7 Conversion of fractions to decimals

003.8 Operations (addition, subtraction, multiplication, division)

004 Ratio, proportion, percentage

004.1 Equivalence of ratios, proportions

004.2 Conversion: percentage, fraction

005 Number theory

005.1 Divisibility tests

005.2 Primes and composites

005.3 Prime factorization

005.4 Relation between divisors, factors, multiples

005.5 Greatest common divisor (gcd) and least common multiple (lcm)

006 Powers and exponents

006.1 Base, exponent, power

006.2 Powers of ten, scientific notation

007 Other numeration systems

007.1 Place value

007.2 Counting

007.3 Converting: decimal and non-decimal

007.4 Operations in binary system

008 Square roots

008.1 Definition

008.2 Estimations

008.3 Reading table of square roots

008.4 Calculation

009 Dimensional analysis (denominate numbers)

009.1 Addition, subtraction, multiplication and division of denominate numbers

009.2 Units associated with area, volume

009.3 Units associated with speed, pressure, etc

100 Algebra

101 Integers

101.1 Extension of whole numbers to integers, number line

101.2 Order

101.3 Absolute value

101.4 Operations, extension to negative integers

101.5 Properties of operations

102 Rationals

102.1 Positive - negative

102.2 Graphical representation

102.3 Order

102.4 Density

102.5 Equivalence

102.6 Operations

102.7 Properties of operations

103 Integer exponents

103.1 Extension of exponents to integers

103.2 Laws of exponents

104 Formulas and algebraic expressions

104.1 Variables

104.2 Algebraic expressions

104.3 Evaluation of formulas or algebraic expressions

104.4 Derivation of formulas and expressions from given information

105 Polynomials and rational expressions

105.1 Polynomials

105.2 Simplification

105.3 Operations: +, -, x

105.4 Special forms $(x + y)^2$, x^2-y^2,

105.5 Factoring

105.6 Rational expressions

106 Equations and inequations

106.1 Linear equations

106.2 Quadratic equations

106.3 Rational equations

106.4 Algebraic, (contains roots of variable) equations

106.5 Solution of linear inequations

106.6 Equations and inequations involving absolute value

107 Relations and functions

107.1 Representation of relations: formula, table, graph, sets of ordered pairs

107.2 Single-valued relations - functions

107.3 Describing functions by rules, constant, linear, quadratic functions

108 Systems of linear equations

108.1 Interpreting information as linear equations

108.2 Graphical and algebraic solution: elimination, substitution, Cramer's Rule

108.3 Inconsistent systems

108.4 Dependent systems

109 Finite systems

109.1 Modular arithmetic tables

109.2 Comparative structure (commutative, associative, etc.)

110 Finite sets

110.1 Specifying a set

110.2 Operations and relations on sets: $\in, \subset, \cup, \cap$, complementation, differences

110.3 Representation of sets

110.4 Properties of operations

111 Flowcharts and programming

111.1 Make flowcharts

111.2 Interpret flowcharts

111.3 Making algorithms

111.4 Programming

112 Real numbers

112.1 Decimal representation

112.2 Operations with irrational numbers

200 Geometry

201 Classification of plane figures: number of edges, parallel sides

202 Properties of plane figures.

203 Congruence of plane figures

203.1 Classification by congruence

203.2 Properties of congruent figures

204 Similarity of plane figures

204.1 Classification

204.2 Recognition

204.3 Scale drawings

205 Geometric constructions

205.1 Ruler and compass

205.2 Mira construction (Mirror Cards)

206 Pythagorean triangles

206.1 Using Pythagoras' theorem

206.2 Pythagorean triples

206.3 Irrational numbers as lengths of hypotenuse

207 Coordinates

207.1 Location of points

207.2 Graphing relations

208 Simple deductions

209 Informal transformations in geometry

210 Relationships between lines and planes in space

211 Solids (symmetry properties)

212 Spatial visualization and representation

213 Orientation (spatial)

213.1 In plane

213.2 In space

214 Decomposition of figures

300 Probability and Statistics

301 Data collection

302 Organization of data

303 Representation of data

304 Interpretation of data

- 304.1 Central tendency
- 304.2 Dispersion
- 304.3 Percentiles

305 Combinatorics

306 Outcomes, sample spaces and events

307 Counting of sets, $P(A \cap B)$, $P(A \cup B)$, independent events

308 Mutually exclusive events

309 Complementary events

400 Measurement

401 Standard units of measure

- 401.1 System of units
- 401.2 Relation between units
- 401.3 Appropriate units

402 Estimation

403 Approximation: precision, accuracy

404 Determination of measures: areas, volumes, etc.

Outline: Population B

Content Dimension for Population B

1 Sets, Relations and Functions

- 1.1 Set notation
- 1.2 Set operations (e.g. union, inclusion)
- 1.3 Relations
  - 1.3.1 Definitions and representations
  - 1.3.2 Properties (symmetric, etc.)
  - 1.3.3 Operations on relations (inverse and composition)
- 1.4 Functions
  - 1.4.1 Definitions and representations, connections with relations
  - 1.4.2 Properties (1-1, onto, etc.)
  - 1.4.3 Operations on functions (inverse, composition)
- 1.5 Infinite sets, cardinality and cardinal algebra (rationals and reals)

2 Number Systems

- 2.1 Common laws for number systems (e.g. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$)
- 2.2 Natural numbers
  - 2.2.1 Induction, recursive definitions
  - 2.2.2 Elements of number theory
- 2.3 Decimals
  - 2.3.1 Finite decimals
  - 2.3.2 Approximation by finite decimals
  - 2.3.3 Infinite decimals, periodicity
- 2.4 Real numbers
  - 2.4.1 Arithmetic of radicals

2.4.2 Powers, with rational exponents

2.4.3 Least upper bound and greatest lower bound

2.5 Complex numbers

2.5.1 Addition and subtraction

2.5.2 Geometric representation

2.5.3 Absolute value and argument

2.5.4 Polar representation

2.5.5 Multiplication and division

2.5.6 De Moivre's formula

3 Algebra

3.1 Polynomials (over R)

3.1.1 Addition and multiplication

3.1.2 Factorization

3.1.3 Evaluation, roots or zeros

3.1.4 Degree - number of roots (or zeros)

3.1.5 Quadratic equations

3.1.6 Polynomial equations of degree > 2

3.2 Quotients of polynomials

3.2.1 Addition, subtraction, multiplication and division.

3.2.2 Partial fractions (eg $\frac{2x-1}{x^2-3x+2} = \frac{3}{x-2} - \frac{1}{x-1}$)

3.3 Roots and radicals

3.3.1 Simplification (e.g. $\sqrt{a^2b} = a\sqrt{b}$)

3.3.2 Rationalizing denominators (e.g. $\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{a-b}$)

3.4 Equations and inequalities

3.4.1 Linear equations

3.4.2 Quadratic equations

3.4.3 Polynomial equations of degree > 2

3.4.4 Linear inequalities

3.4.5 Quadratic inequalities

3.4.6 Inequalities of quotients (e.g. $\frac{x}{x-1} < 0$)

3.4.7 Inequalities and equations involving absolute value (e.g. $|3x + 2| + |2x - 1| < 4$)

3.5 Systems of equations and inequalities

3.6 Matrices and determinants

3.7 Groups, rings and fields

3.7.1 Definitions and examples

3.7.2 Properties

4 Geometry

4.1 Euclidean (synthetic) geometry

4.1.1 Informal treatment

4.1.2 Axiomatic treatment - Euclid's axioms

4.1.3 Axiomatic treatment - transformations

4.2 Affine and projective geometry in the plane

4.3 Analytic (coordinate) geometry in the plane

4.3.1 Lines: slope, equations of, parallel, perpendicular, families

4.3.2 Distance: between points, from point to line

4.3.3 Conics: definitions, equations, properties

4.4 Three-dimensional coordinate geometry

4.5 Vector methods

4.6 Trigonometry

4.6.1 Right angled triangles

4.6.2 General triangles

4.6.3 Trigonometric identities (formulas)

4.7 Finite geometries (e.g.projective or affine geometry with a finite set of points)

4.8 Elements of topology (e.g.Euler's formula for polyhedra)

5 Analysis

5.1 Elementary functions

5.1.1 Polynomial functions

5.1.2 Rational functions

5.1.3 Algebraic functions (e.g.$f(x) = \sqrt{x}$)

5.1.4 Circular functions

5.1.5 Inverse circular functions

5.1.6 Exponential functions

5.1.7 Logarithmic functions

5.2 Properties of functions

5.2.1 Domain and range

5.2.2 Composition

5.2.3 Inverse

5.3 Limits and continuity

5.3.1 Definition and examples

5.3.2 Theorems (e.g.limits of sums of functions, of products, of quotients)

5.3.3 Asymptotes of functions

5.3.4 Limits of sequences and series of constants

5.4 Differentiation

- 5.4.1 Definition and examples
- 5.4.2 Of elementary functions
- 5.4.3 Of a sum
- 5.4.4 Of a product
- 5.4.5 Of a quotient
- 5.4.6 Of a composite (chain rule)
- 5.4.7 Of the inverse of a function
- 5.4.8 Logarithmic differentiation

5.5 Applications of the derivative

- 5.5.1 For finding maxima and minima
- 5.5.2 To graphing of functions
- 5.5.3 Mean value theorem
- 5.5.4 Extended Mean Value Theorem (Taylor's formula)
- 5.5.5 Differentials (approximation)

5.6 Integration

- 5.6.1 Numerical integration - approximating the area under a curve
- 5.6.2 Definite integral
- 5.6.3 Functions defined by integrals (e.g. $\text{Log } x = \int_1^{x} \frac{1}{t}\, dt$)
- 5.6.4 Antiderivative
- 5.6.5 Fundamental theorem of calculus

5.7 Techniques of integration

- 5.7.1 Integration of a linear combination of functions
- 5.7.2 Integration by substitution
- 5.7.3 Integration by parts
- 5.7.4 Integration by partial fractions

5.8 Applications of integration

5.8.1 Areas

5.8.2 Volumes of rotation

5.8.3 Other volumes

5.8.4 Arc length

5.9 Differential equations

5.9.1 Definition and examples

5.9.2 First order linear differential equations

5.9.3 Second order linear differential equations with constant coefficients

5.9.4 Separable equations

5.9.5 Homogeneous equations

5.10 Sequences and series of functions

5.10.1 Power series

5.10.2 Maclaurin and Taylor series

6 Probability and Statistics

6.1 Probability

6.1.1 Sample space (set of possible outcomes)

6.1.2 Probability of an event

6.1.3 Independent events

6.1.4 Conditional probability

6.1.5 Probability theorems (e.g. Bayes' theorems)

6.1.6 Bernoulli trials

6.1.7 Expectation

6.1.8 Markov chains

6.1.9 Random variables

6.1.10 Monte Carlo methods

6.2 Statistics

6.2.1 Data collection and organization

6.2.2 Data representation

6.2.3 Interpretation

- Central tendency
- Dispersion
- Percentiles
- Measures of relationship

6.3 Distributions

6.3.1 Normal distributions

6.3.2 Binomial distributions

6.3.3 Central limit theorem

6.4 Statistical inference

6.4.1 Point estimation

6.4.2 Interval estimation

6.4.3 Sampling distributions

6.4.4 Standard error

6.4.5 Hypothesis testing

- Type I error
- Type II error

6.4.6 Tests of significance

- Z test
- t test
- X^2 test

6.5 Bivariate statistics

6.5.1 Regression

6.5.2 Correlation

7 Finite mathematics

7.1 Combinatorics

7.2 Game theory

7.3 Graphs and networks

8 Computer Science

8.1 Flowcharts

8.2 Computer languages and programming

8.3 Algorithms

8.4 Data processing

9 Logic

9.1 Propositional logic

9.1.1 Boolean algebra

9.1.2 Switching circuits

9.2 Predicate logic

9.2.1 Quantifiers (eg. "there exists", "for all")

9.3 Rules of inference

9.4 Axiomatics; deductive systems

IV Behaviors

In utilizing a classification of behavioral levels to describe objectives, goals, or test items, one has to make hypotheses about the way students will respond to a task, the background the students bring to the task, and what is the main focus of the task. For example, a complex problem solving task may be a simple recall task the second time a student sees it; a task is not complex problem solving at all if the student does not attempt it. Assumptions and hypotheses are implicit in any use of a content × behavior grid. Apparent lack of agreement on how a task might be classified can usually be attributed to different sets of hypotheses or assumptions on the students' background or behavior with respect to the task.

Obviously, the behavioral classification could vary for individual students, but in a descriptive or heuristic use of the grid, such as we have for IEA, "usual", "normal", or "typical" behaviors must be hypothesized.

Example items from this section are taken from Tests A, B, and C of the 1964 IEA mathematics survey and the classification has been done for Population A students.

Discussion of Behaviors Categories

Computation Behaviors

The computation-level behaviors represent the least complex behaviors that we expect from students as outcomes of instruction in mathematics. The computation level should be described so as to include exercises of simple recall and exercises of routine manipulation. The level represents primarily those outcomes that require of the student no decision making or complex memory.

Example items:

43.0-17.6 is equal to ——.

$\frac{2}{5} + \frac{3}{8}$ is equal to

A $\frac{5}{13}$ B $\frac{5}{40}$ C $\frac{6}{40}$ D $\frac{16}{15}$ *E $\frac{31}{40}$

Which of the following is the same as a quarter of a million?

A 25,250 C $\frac{1}{4,000,000}$ *D 250,000

B 40,000 E 2,500,000

One example of a computation-level behavior is the ability to recall specific facts. This could include objectives where the student is expected to reproduce or recognize material in almost exactly the same form as it was presented in the course of study. It could also include fundamental units of knowledge that a student can be reasonably assumed to know because he has been exposed to them over a long period of time. An example of the latter case would be the basic number facts. Presumably, students

have been exposed to the number facts all through the elementary grades; items explicitly designed to measure number facts might not correspond to an explicit part of the course of study. On the other hand, to say that the student is expected to know that the measure of a right angle is 90° is an example of an outcome which assumes that material has been covered in the course of study.

Another example of a computation behavior would be the ability to use mathematics terminology.

For instance, for an appropriate course of study, students should be able to recognise an acute, an obtuse, or a right angle; in an algebra course they should know what is meant by the instruction "simplify the expression."

Obviously, knowledge of specific facts and knowledge of terminology are required as part of any more complex level of behavior. Measuring a more complex behavior, therefore, involves the indirect assessment of behaviors dealing with knowledge of specific facts or knowledge of terminology. There are objectives in the content outline, however, for which we may want to assess these very elementary behaviors.

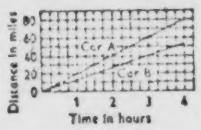
A most important example of the computation behavior is the ability to carry out algorithms. Restated, this is the ability to manipulate elements of a stimulus according to some learned rules. The student is not expected to select the algorithm; such selection involves a certain level of choosing and decision-making which appropriately belongs at a more complex level of behavior. The algorithms are not limited to arithmetic: for instance, bisecting a line segment might be a computation behavior in geometry, and solving a simple linear equation might be a computation behavior in algebra.

Comprehension Behaviors

The comprehension behaviors are more complex than computation. Comprehension includes, and is sometimes used interchangeably with, understanding. One comprehension behavior that provides evidence of understanding is the translation of a task from one form to another. Other examples are given below.

Example items:

Use the graph on the right in answering the two following questions



Three hours after starting, car A is how many miles ahead of car B?

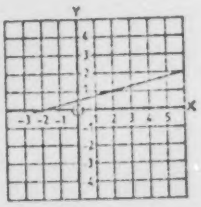
A 2 B 10 C 15 *D 20 E 25

How much longer does it take car B to go 50 miles than it does for car A to go 50 miles?

*A 1 hour 15 minutes
B 1 hour 30 minutes
C 2 hours
D 2 hours 30 minutes
E 2 hours 35 minutes

The equation of the line shown in the graph is

A. $x + 4y = 4$
B. $2x - y = 4$
C. $2x = y - 2$
*D. $x - 4y + 2 = 0$
E. $4x - y = 2$



Which of the following operations with whole numbers will *always* give a whole number?

I Addition
II Multiplication
III Division

A I only
B II only
C III only
*D I and II only
E II and III only

The dividing line between the computation and the comprehension categories is artificial and computation-level behaviors are sometimes assumed or incorporated within comprehension-level behaviors. One example of a comprehension behavior is the ability to recognise mathematical concepts. This is included as comprehension behavior because a concept is an abstraction, and an abstraction theoretically requires some implicit decision-making in using a concept or in saying whether an object is an instance of a concept. In some sense, the knowledge of a concept is more complex than the knowledge of a specific fact.

Similar reasoning applies to the ability to recognise mathematical principles, rules, and generalizations. Hence, such behaviors are included under comprehension. Such behaviors assume a correspondence with a course of study. Whether an item measures knowledge of a principle, rule, or generalization depends upon the material the student has studied. If he must generate a principle, rule, or generalization, or use a principle, rule, or generalization in order to answer a question, then the behavior is at a higher level than comprehension.

One central comprehension-level behavior, mentioned above, is the ability to transfer problem elements from one mode to another. This can mean translation from a verbal description to a pictorial representation, or translation from a verbal representation to a symbolic form, or vice versa in each case.

Another example of comprehension-level behavior is the ability to follow a line of reasoning. This might be restated as the "ability to read or listen to a mathematical argument." It is the ability to receive communication about mathematics. Likewise, the ability to read and interpret a mathematics problem is also an example of comprehension-behavior. This is far less than the ability to solve a problem, but it is certainly a necessary step. In the reading of mathematics material and problems, there appear to be skills involved that go beyond general verbal skills and reading abilities.

Application Behaviors

The application behaviors involve a sequence of responses from the student; this characteristic distinguishes them from computation-level or comprehension-level behaviors. On the other hand, the application-level behaviors are to be closely related to the course of study: they deal with activities that are routine in the sense that items like these application-level items(not identical to them) would have been studied. The transfer to new situations is minimal.

Example items:

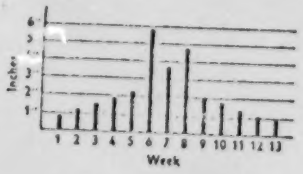
How many seven-man teams can you make out of 7 nine-man teams?

A 7 B 8 *C 9 D 16 E 63

There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total membership of the sports club? Ans. 143

The lengths of the sides of a triangle XYZ are 4, 7, and 10. If a similar triangle has a perimeter of 147, what is the length of its shortest side? Ans. 28

In the graph on the right, rainfall in inches is plotted for 13 weeks. The average weekly rainfall during the period is approximately



A 1 inch
*B 2 inches
C 3 inches
D 4 inches
E 5 inches

Four examples of application behaviors will help further illustrate this category. First, the ability to solve routine problems, in its most limited case, involves selecting and carrying out an algorithm. If the problem is stated

verbally, the behavior of solving is preceded by the behavior of formulating the problem in symbolic terms. The sequence can be more complicated, for the problem may involve selecting a principle or rule, using the principle in selecting an algorithm or making several calculations. If it cannot be expected that the student will recognize the problem as similar to those he has studied in the course of instruction, then higher-order problem-solving is required and the item would be at the analysis level.

The ability to make comparisons is an application-level behavior because the student is expected to recall relevant information (concepts, rules, mathematical structure, terminology), discover a relationship, and formulate a decision. When a student is making comparisons, he is in a sense generating an algorithm and then following the algorithm to make a decision. This generation is of a routine nature and therefore belongs at the application level rather than at the analysis level. An important part of this ability is the behavior of choosing from among available alternatives.

The ability to analyze data is the third applications example. This is a well-practiced part of the mathematics curriculum. It involves reading and interpreting information, manipulating that information, and making decisions or drawing conclusions as a result. The behavior is the ability to separate a problem into its component parts, to distinguish relevant from irrelevant information, and to establish a connection with subproblems which have already been solved.

The ability to recognize patterns, isomorphisms, and symmetries may involve recalling relevant information, transforming problem elements, manipulating these elements in a sequence, and recognizing a relationship. It is a sequence of behaviors and hence belongs rightfully at the application level. If the student is asked to formulate or generate new patterns, isomorphisms, or symmetries, then the behavior belongs at the analysis level. It is assumed that similar patterns, isomorphisms, or symmetries have been studied by the student and that recognition is possible; otherwise, a higher-level behavior is implied.

Analysis Behaviors

This behavior level is the highest of the cognitive categories - comprising the most complex behaviors. It includes most behaviors described in the Taxonomy* as analysis, synthesis, or evaluation. It includes what Avital and Shettleworth** have called "open search." Here we will include nonroutine problem-solving, discovery experiences, and creative behavior as it relates to mathematics.

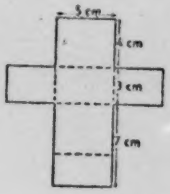
* Bloom, B S (Ed) Taxonomy of educational objectives: The classification of educational goals. Handbook 1 Cognitive domain. New York: McKay, 1956.

** Avital, S M & Shettleworth, S J Objectives for mathematics learning (Bull No 3) Toronto, Ontario, Can: Ontario Institute for Studies in Education, 1968.

It differs from application-level or comprehension-level behaviors in that it involves a degree of transfer to a context in which there has been no practice. In order to respond satisfactorily to test items at this level, there must be a greater reliance on heuristic behavior. Many of the "ultimate" objectives of mathematics instruction are at the analysis level.

Example items:

A piece of tin with dimensions as shown is to be folded along the dotted lines to make a box. What is the volume, in cubic centimeters, enclosed in the box? Ans. 60



Three straight lines intersect as shown in the figure on the right. What is x equal to in degrees?

A 30 B 50 C 60 *D 110 E 150

One bell rings every 8 minutes, while another bell rings every 12 minutes. They have rung together once at the same moment. After how many minutes will they ring together again

A for the first time? Ans. 24

B for the second time? Ans. 48

C for the tenth time? Ans. 240

The following include some examples of analysis behaviors. First, the ability to solve nonroutine problems requires the student to exhibit transfer of previous mathematics learning to a new context. Perhaps some word other than "nonroutine" would be more descriptive; the objective is to develop the ability to solve problems unlike those which have been solved previously. Such problem-solving may involve separating a problem into parts and exploring what can be learned about each part. It may involve reorganizing the problem elements in a new way in order to determine a solution. In all cases, the student is given a problem situation for which an algorithmic solution is not available to him. Perhaps a heuristic approach is required - such as establishing a plan and carrying it out, or repeatedly

comparing the given situation and the goal to determine differences, which are in turn eliminated one at a time to gradually bring about a solution.

Another analysis-level example is the ability to discover relationships. To discover requires the restructuring of problem elements in a new way to formulate a relationship. This ability differs from the last category of the applications level in that the student must discover (formulate) a new relationship rather than recognize a familiar relationship in new data.

The ability to construct proofs is an essential analysis-level behavior. The language of proof is the language by which the mathematician presents his work to his colleagues. Here, we are interested in the ability to construct proofs as opposed to the ability to reproduce proofs (application-level) or recall proofs (computation-level). Thus, when the proof of the theorem that $\sqrt{2}$ is irrational has been developed in class and then the item "Prove $\sqrt{3}$ is irrational" is assigned, most likely the proof will correspond exactly to the proof dealing with $\sqrt{2}$ and therefore will exhibit application-level behavior.

The ability to criticize proofs is an analysis-level behavior. Perhaps this could be stated a little more generally as the "ability to criticize any mathematical argument."

Finally, the ability to formulate and validate generalizations is included as an analysis behavior. It is similar to some previous analysis categories; stated another way, it is the ability to discover a relationship and to construct a proof to substantiate the discovery.

Outline of the

Behaviors Dimension for Tables of Specification

Computation Behavior

Examples:

Ability to recall specific facts
Ability to use mathematics terminology
Ability to carry out algorithms

Comprehension Behavior

Examples:

Ability to recognize concepts
Ability to recognize mathematics principles, rules and generalizations
Ability to transform problem elements from one mode to another
Ability to follow a line of reasoning
Ability to read and interpret a problem

Application Behavior

Examples:

Ability to solve routine problems
Ability to make comparisons
Ability to analyze data
Ability to recognize patterns, isomorphisms, and symmetries

Analysis Behavior

Examples:

Ability to solve nonroutine problems
Ability to discover relationships
Ability to construct proofs
Ability to criticize proofs
Ability to formulate and validate generalizations

V Examples

The appropriate use of a content × behaviors grid is heuristic and descriptive rather than prescriptive.

Clearly, for a level of behavior to be measured by an item an assumption must be made about the background of students and the manner in which they may respond to an item.

Illustrations, and applications, must necessarily be for expectations of "most students" or the "average" students. The chapter by Wilson* provides abundant illustrations of different content items at each of the behavior levels. In what follows, the items from the 1965 IEA mathematics tests have been classified in the grids as an illustration. The test booklets for the 1965 IEA Mathematics Study are reproduced as Appendix II. (Please note these pages 84-95, have not been numbered.)

* Wilson J W ; Evaluation of Learning in Secondary School Mathematics; In (Ed Bloom B S, Hastings J T, Madaus G F; Handbook on Formative and Summative Evaluation of Student Learning, McGraw-Hill, 1971)

Classification of TEST ITEMS: 13-YEAR-OLDS, 1965*

	Computation Behavior	Comprehension Behavior	Application Behavior	Analysis Behavior
000 Arithmetic	A.1, A.3, A.6, A.9	A.4, A.17 A.23	A.2, A.5 A.18	
	B.1, B.3, B.12	B.5, B.16, B.17 B.21	B.4, B.6 B.7	B.8
	C.1, C.2, C.3	C.17		C.23
100 Algebra	A.12, A.14	A.11	A.15, A.20	
	B.10, B.11, B.15	B.9, B.20, B.22 B.23, B.24	B.2, B.14	
	C.9, C.18	C.16	C.4, C.6 C.19, C.21	C.10
200 Geometry			A.19	A.10 A.16 A.21
				B.18
		C.12, C.13, C.14	C.11	C.20, C.22
300 Probability and Statistics				
400 Measurement	C.8		A.7, A.8, A.22	A.13
			B.19	

*Entries in the table, eg C.8, refer to the test and item number in the 1965 IEA Battery. Hence, "C.8" would mean "Test C, item 8."

IEA MATHEMATICS STUDY

TEST ITEMS: PRE-UNIVERSITY, 1965*

	Computation Behavior	Comprehension Behavior	Application Behavior	Analysis Behavior
1.0 Sets, Relations & Functions	7.3, 8.13	6.13	7.4, 7.7, 7.10	8.9, 8.5 8.7, 8.8
2.0 Number Systems	5.15 7.8	5.17		
3.0 Algebra	5.1, 5.8, 5.20, 7.1, 7.2, 7.9, 7.15, 8.6	5.2, 5.3, 5.4, 5.5, 5.6, 6.7, 7.12, 7.13 8.2	6.2, 6.16 7.5, 7.6, 7.17	5.14 6.4, 7.15 7.11, 7.14 8.14
4.0 Geometry	9.13	5.7, 5.9 6.9, 6.14 9.1	5.8, 5.21 6.1, 6.3, 6.5, 9.14	8.15 6.8, 6.11 6.17, 8.4
5.0 Analysis	5.16, 6.15, 9.5, 9.15	5.10, 5.11, 5.19 8.11, 8.16, 9.8, 9.10, 9.11	5.12, 5.13, 9.12, 9.7 6.10, 8.1 8.10. 9.3 9.4, 9.6	8.12 9.2 9.9
6.0 Probability & Statistics			6.6, 6.12 8.3	
7.0 Finite Mathematics				
8.0 Computer Science				
9.0 Logic				
10.0 History of Mathematics				

*An entry in the table refers to the 1965 IEA Mathematics Test and item number. Eg, "5.16" means "test 5, item 16."

APPENDIX I

DESCRIPTION OF THE

POPULATION A

CONTENT OUTLINE

001 Natural Numbers and Whole Numbers

001.1: Reading and writing large numbers

--representation of numbers in terms of base 10 place value numeration system.

001.2: Operations with whole numbers

--the standard algorithms for adding, subtracting, multiplying and dividing numbers of several digits.

--Operations as functions:

--Functions defined in terms of the operations

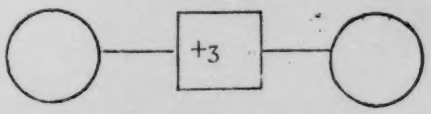
e.g., addition of a fixed number

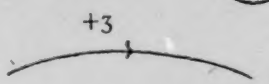
multiplication by a fixed number

the special role of addition of 0 and of multiplication by 1.

--Representation of functions: as machines

e.g.,



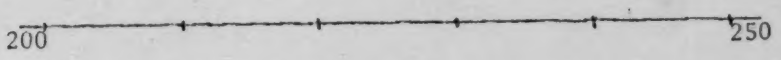


: as arrows

: on number line

001.3: Representing whole numbers on a number line--extending the number line:

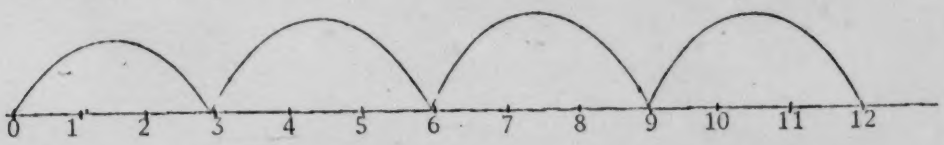
e.g.,



--scales on measuring instruments

--translations along number line adding a fixed number

--"stretching" number line: multiplication by a fixed number (representation by "hops" on number line)



4 x 3 = 12

001.4: Properties of the operations--commutativity and associativity of addition and multiplication

--distributivity.

001.5: Ordering whole numbers--comparing whole numbers, especially large numbers.

--significance of place values in ordering numbers

--properties of the order relation: transitivity

--monotonicity of functions "addition of a fixed number"; "multiplication by a fixed number (>0)", e.g.,

$a < b \Rightarrow ac < bc \quad (c > o)$

$a < b \Rightarrow a + c < b + c$

001.6: Estimating--estimating sums, differences, products and quotients.

e.g. (1) How many bars of chocolate at 14¢ each, can we buy with $2.00?

(2) The same soap bars are on sale at one store at 4 for 23¢ and at another store at 3 for 19¢. Which is cheaper?

(3) Which is more expensive,

3 for 23¢ or 4 for 19¢?

--estimating in long division

--estimating intervals: e.g., If x lies between 6.95 and 7.05, between what values does $8x + 4$ lie?

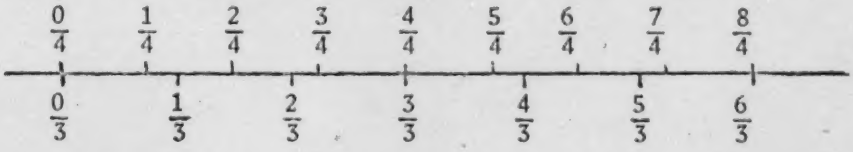
002 Fractions (for non-negative rational numbers)

002.1: The description of various situations by means of fractions

002.2: Comparison of two fractions--equivalent fractions

--ordering of rational numbers represented by fractions

002.3: Representation of fractions as points on a number line.



--as operators

--as magnitudes in terms of a given unit.

002.4: Simplification of fractions--the reduction of a given fraction, i.e., the determination of a fraction equivalent to a given fraction but with smaller (reduced) numerator and denominator.

--the converse; determination of a fraction equivalent to a given fraction but with denominator a multiple of the original one..

002.5: Mixed numerals--representation of a rational number as the sum of a whole number and a fraction < 1. e.g., 3 1/4.

--Improper fractions: where numerator exceeds denominator, e.g., 13/4

--conversion of improper fractions to mixed numerals (13/4 → 3 1/4)

--conversion of mixed numerals to improper fractions (3 1/4 → 13/4)

002.6: Density: Determination of a rational number between two given rational numbers. e.g., given 2/3 and 3/4 then find x (say 17/24) a rational number such that $2/3 < x < 3/4$

002.7: Operations: --addition and subtraction of rational numbers represented by fractions

--multiplication and division of rational numbers represented by fractions

002.8: Approximation of rational numbers:

--approximation of a rational number represented by a fraction by one represented by a simpler fraction

e.g. $\frac{236}{475} \doteq \frac{1}{2}$

002.9: Estimation: --estimating the result of specific operations on specified rational numbers. e.g.

$$\frac{236}{275} \times \frac{124}{503} \doteq \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.$$

003 Decimal Fractions

003.1: Rational numbers represented by fractions with denominators a power of 10 --decimal fractions

003.2: Place value for decimal fractions

003.3: Order of decimal fractions

e.g. Which is larger

236.327 or 236.356 ??

003.4: Representation on number line

e.g.

003.5: Representation of rational numbers by decimal fractions

--as decimal (finite) fractions, e.g. 1/4 = 0.25

--as repeating decimals: e.g., 1/3 ≑ 0.33

--periodicity

003.6: Approximation of rational numbers by decimal fractions.

e.g., $1/3 = 0.33$

--estimation of error (e.g., $1/3 > 0.33 = 1/300$)

--rounding off, e.g. 0.467 921 4

$\doteqdot$ 0.468

003.7: Conversions - conversion of fractions to decimals

--conversion of decimal fractions and periodic decimals to fractions.

003.8 Operations:

--addition and subtraction of positive rational numbers represented by decimal fractions

--multiplication and division of positive rational numbers represented by decimal fractions.

004 Ratio, Proportion, Percentage

004.1: Equivalence of ratios

e.g., 2:3 = 4:6

Equivalence of proportions

e.g., 2:3:4 = 4:6:8

004.2: Representation of a rational number by a fraction with denominator 100, i.e, percentage

--conversion of fractions to percent, e.g., 3/4 = 75%

--conversion of percent to fractions

005 Number Theory

005.1: Divisibility tests-divisibility by 2 (unit digit 0, 2, 4, 6, 8)

-divisibility by 3 (or 9) [sum of digits divisible by 3 (or 9)]

005.2: Prime numbers, composite numbers

--definition of prime numbers

--determination of prime numbers < 100

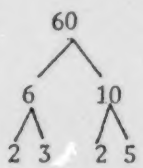
005.3: Prime factorization--determining the factors of a whole number

e.g., set of factors of 12 {1, 2, 3, 4, 6, 12}.

--factors of a factor, e.g., 3 is a factor of 6, 6 is a factor of 48; therefore 3 is a factor of 48.

--factors from factors, e.g., 2 is a factor of 30, 3 is a factor of 30 therefore 6 is a factor of 30.

--factor trees



--factorizations of a number

e.g., 60 = 6 x 10 = 5 x 12.

--prime factors, prime factorization

e.g., prime factors of 60 are 2, 3, and 5

prime factorization of 60 = 2 x 2 x 3 x 5.

--sets of factors

--sets of multiples

--uniqueness of prime factorization

005.4 Relation between divisors, factors and multiples:

e.g., a divides b if and only if a is a factor of b

if and only if b is a multiple of a.

005.5: Greatest Common Divisor (gcd) and Least Common Multiple (lcm)

--definition

--determination of gcd and lcm by factoring

--relative prime numbers

--prime powers: using prime powers in factorizations

006 Powers and Exponents

006.1: Bases and exponents--definition

--evaluation of powers

--squares, cubes, e.g., given a cube of edge a given length,

what happens to the volume when the length of the edge is doubled?

006.2: Powers of 10, Scientific notation

e.g., $6\ 700\ 000 = 6.7 \times 10^6$

--negative exponents

--multiplication and division of powers of 10

--order of magnitude

--multiplication and division of numbers in scientific notation

007 Other Numeration Systems

007.1: Place value - the "value" of a digit in other bases

e.g., $435_{six} = 4 \times 6^2 + 3 \times 6 + 5$

007.2: Counting - What is the next number after 435_{six} ? (440_{six}).

007.3: Conversion - changing from one base to another

$435_{six} = 167$

007.4: Binary System -- even and odd in the binary system

--addition, subtraction, multiplication and division in the binary system

008 Square Roots

008.1: Definition

008.2: Estimation - e.g. $\sqrt{15}$ is less than 4

$\sqrt{72} \doteq 6 \times 1.4$ ($\sqrt{2} \doteq 1.4$)

008.3: Reading table of square roots

008.4: Calculation of square roots

e.g., $\sqrt{12 \times 75} = \sqrt{4 \times 3 \times 3 \times 25} = 2 \times 3 \times 5$

009 Dimensional Analysis (denominate numbers)

009.1: Addition of denominate numbers

e.g., 30 cm + 20 cm = 50 cm

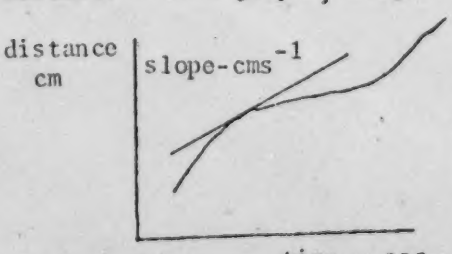
30 cms^{-1} + 20 cms^{-1} = 50 cms^{-1}

Multiplication and division of denominate numbers

e.g., 30 cm x 5 cm = 150 cm^2

30 cm ÷ 20s = 3/2 cms^{-1}

e.g., distance - time graph , slope



009.2: Units associated with area, volume

009.3: Units associated with speed, pressure, etc., e.g, cms^{-1}, Newtons/m^2

--effect of change of force in pressure, say

--effect of increasing speed, etc.

101 Integers

101.1: Extension of whole numbers to integers

--introduction of negative integers

--extension of number line to include negative integers

101.2: Order: --extending the order relation to negative integers

e.g., Is -24 greater or less than -30?

101.3: Absolute value --e.g., $|-24| = 24$.

--Order these: -24, 4, -30; from greatest to least.

--Order these: $|-24|$, $|4|$, $|-30|$; from greatest to least.

101.4: Extension of the operations to negative integers--addition, subtraction, multiplication and division of integers

--closure of subtraction in the set of integers

101.5: Properties of operations--commutativity and associativity of addition

--commutativity and associativity of multiplication

--distributivity

--involutivity of oppositing

e.g., $-(-\bar{4}) = \bar{4}$ [or $-(-(-4)) = -4$]

--Relation between "multiplying by $\bar{1}$"
and oppositing. e.g., $-2 = \bar{1} \times 2$

--non-commutativity of subtraction

e.g., $a-b = -(b-a)$

--non-associativity of subtraction

$a - (b - c) \neq (a - b) - c$ in general

--the function "subtraction of a" as inverse of the function "addition of a".

e.g., $(5 - 4) + 4 = 5$

$(5 + 4) - 4 = 5$.

102 Rationals

102.1: Extension to negative rationals

--introduction of negative rationals

e.g., $-2/3 + 2/3 = 0$

--relation of rationals to integers

e.g., $\bar{2}/3 = 2/\bar{3} = -2/3$

102.2: Graphical representation --points on a number line

--slopes, e.g., $-2/3$ represented by line through origin and point $(3, -2)$.

102.3: Order relation - extension of the order relation to negative rationals.

102.4: Density--determination of a rational number between two given distinct rationals, e.g., between - 3/4 and - 1/3

102.5: Equivalence of fractions - integer fractions: e.g., -2/3
--equivalence of fractions, e.g., -4/6 = - 2/3

102.6: Operations -- extension of the operations to negative fractions;

102.7: Properties of operations--commutativity and associativity of addition
--commutativity and associativity of multiplication
--distributivity
--reciprocation: e.g., the reciprocal or inverse of -3/4 is 4/-3 = - 4/3
--"multiplication by the reciprocal of the (non-zero) rational" as the inverse of the function "multiplication by a (non-zero) rational."
--involutivity of reciprocation e.g., the reciprocal of the reciprocal of a rational number is that rational number.

103 Integer Exponents

103.1: Extension of exponents (with non-zero base) from natural numbers to whole numbers and integers (0 and negative integers).

e.g., $3^0 = 1$

$(4^{-2}) = 1/4^2$

103.2: Laws of exponents

-- $a^n \cdot a^m = a^{n+m}$

-- $a^n / a^m = a^{n-m}$

-- $(ab)^n = a^n b^n$

-- $(a^n)^m = a^{nm}$

-- $a^{-1} = 1/a$, $(a^n)^{-1} = 1/a^n$

104 Formulas and Algebraic Expressions

104:1 Variables as placeholders for numbers--the use of variables in formulas to describe specific operations on numbers, e.g.,

The measure of the volume of a cylinder (in cm^3) is π times the measure, (in cm), of the height multiplied by the (square of the) measure of the radius of the base (in cm)

$V = \pi h r^2$

104.2: Algebraic expressions--as recipes or functions: e.g. "add four to three times the number" = "3x + 4"

104.3: The evaluation of formulas or algebraic expressions-substituting numbers for variables in formulas or algebraic expressions: e.g., the nth triangular number $T_n = \frac{n(n+1)}{2}$. What are the 5th and 10th triangular numbers?

--substituting algebraic expressions for variables in algebraic expressions or formulas. e.g.

Evaluate $a^2 + b^2$ where

$a = u^2 - v^2$

$b = 2uv$

104.4: The use and derivation of a formula or algebraic expression to describe given information: e.g., The annual base rent of an apartment is 85¢ per square metre of floor space. In addition, every rentor must pay a fixed fee of $25 every year. Express the rent per month (in dollars) in terms of the measure of the area (in square metres) of the apartment, i.e., write a formula for the monthly rent of an apartment (in dollars) when the measure of the area (in square metres) is given. Determine the monthly rental of an apartment of area 110 m^2

105 Polynomials and Rational Expressions

105.1: Polynomials--polynomial functions as the performance of operations of arithmetic on a variable regarded as pronumeral or place-holder for a number.

e.g., "$3x^2 + 2x + 4$" is an expression for "three times the product of a number with itself, increased by 4 more than twice the number."

In this way, polynomials may be regarded as an "extension" of the number system.

105.2: Using properties of the operations for simplifying polynomials

e.g., $3\ (4x + 2) = 12x + 6$

using the distributive property.

105.3: Operations--extending operations to polynomial expressions

--addition and subtraction of polynomials using extended properties of the operations, e.g.,

$$(3x^2 + 2x + 4) + (3x + 7) = 3x^2 + 5x + 11$$

--multiplication of polynomial expressions by regarding variables as numbers with respect to the operations.

--use of various properties of the extended operations, e.g.,

$$(x + y)\ \ (3x^2 + 2xy + y^2)$$

$$= (x + y)\ (3x^2) + (x + y)(2xy) + (x + y)y^2$$

(distributivity)

105.4: Special forms:

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$x^2 - y^2 = (x + y)(x - y)$$

105.5: Factoring polynomials

105.6: Rational expressions

-- e.g., $\frac{3x + 4}{2x + 1}$

106 Equations and Inequations (Inequalities)

106.1: Linear equations: Solution of linear equations

e.g., $3x + 2 = 8$

or $3x + 2 = 4x - 7$

106.2: Quadratic equations -- solutions of quadratic equations

106.3: Rational equations -- solution of equations involving rational expressions

e.g., $\frac{2x - 1}{x + 2} + \frac{x - 4}{x - 3} = \frac{x}{x - 3}$

106.4: Algebraic equations - solutions of equations involving square roots of polynomial expressions.e.g. $\sqrt{x^2 - 3x + 2} = x + 2$

106.5: Linear inequations (inequalities) -- solving linear inequalities

e.g.: What is the solution set of $5x + 5/3 > -4/3$?

106.6: Equations and inequations involving absolute value - solution of equations of the form

$$|3x + 2| = 4$$

--solution of equations of the form

$$|2x + 3| = |4x - 1|$$

--solutions of inequations of the form

$$|2x + 3| < 4$$

--solutions of inequations of the form

$$|2x + 3| < |4x - 1|$$

107 Relations and Functions

Relations are here regarded as sets of ordered pairs (as distinct from how they may be discussed in the classroom).

107.1: Representation of a relation

--by a formula

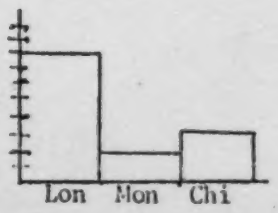
e.g., $a < b$ if for some $c > o$, $a + c = b$

--by a table

Populations of cities

Chicago	3 500 000
Montreal	2 000 000
London	8 000 000

--by a graph



bar graph

--by a pictograph

Chicago 웃웃웃웃

Montreal 웃웃

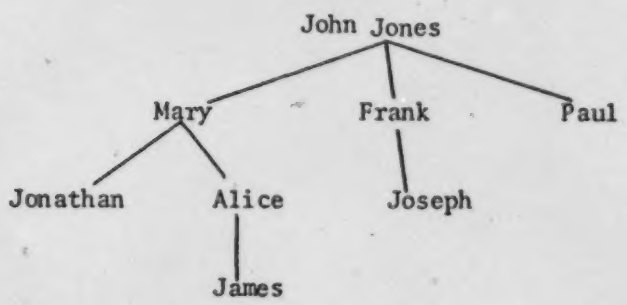
London 웃웃웃웃웃웃웃웃

1 symbol represents 1 million people

--as a set of ordered pairs

$\{(x, y): \quad x^2 + y^2 = 1\}$

--arrow graphs, e.g., family tree



107.2: Single-valued relations--functions

107.3: In the decimal representation of a real number, delete all digits after the first digit right of the decimal point; however, if the second digit right of the decimal point is 5 or greater than 5, add 0.1 to the result

e.g. 3.415. . . $\longrightarrow$ 3.4

3.476. . . $\longrightarrow$ 3.4 + 0.1 = 3.5

3.97621 $\longrightarrow$ 3.9 + 0.1 - 4.0

--constant functions

e.g., the function "multiplication by zero" graph of a constant function

--linear functions

e.g., "add 4 to 3 times the number"

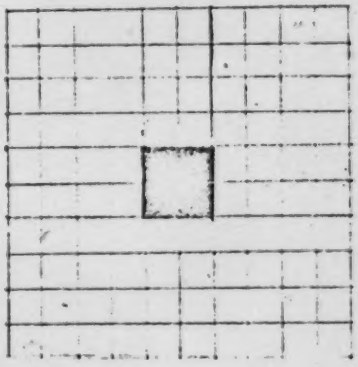
$x \longrightarrow 3x + 4$

graph of a linear function

--quadratic functions

: representing information

e.g., a square is made up of small square tiles in the pattern of a centre block of 4 black tiles surrounded by a uniform border of white tiles. Express the number of white tiles in terms of the number of tiles along an edge of the square



$n \longrightarrow n^2 - 4 \qquad n \geq 2$

108 Systems of Linear Equations

108.1: Describing information in terms of systems of linear equations.

e.g., I have one dollars worth of nickles and dimes but 2 more nickles than dimes

x = number of nickles

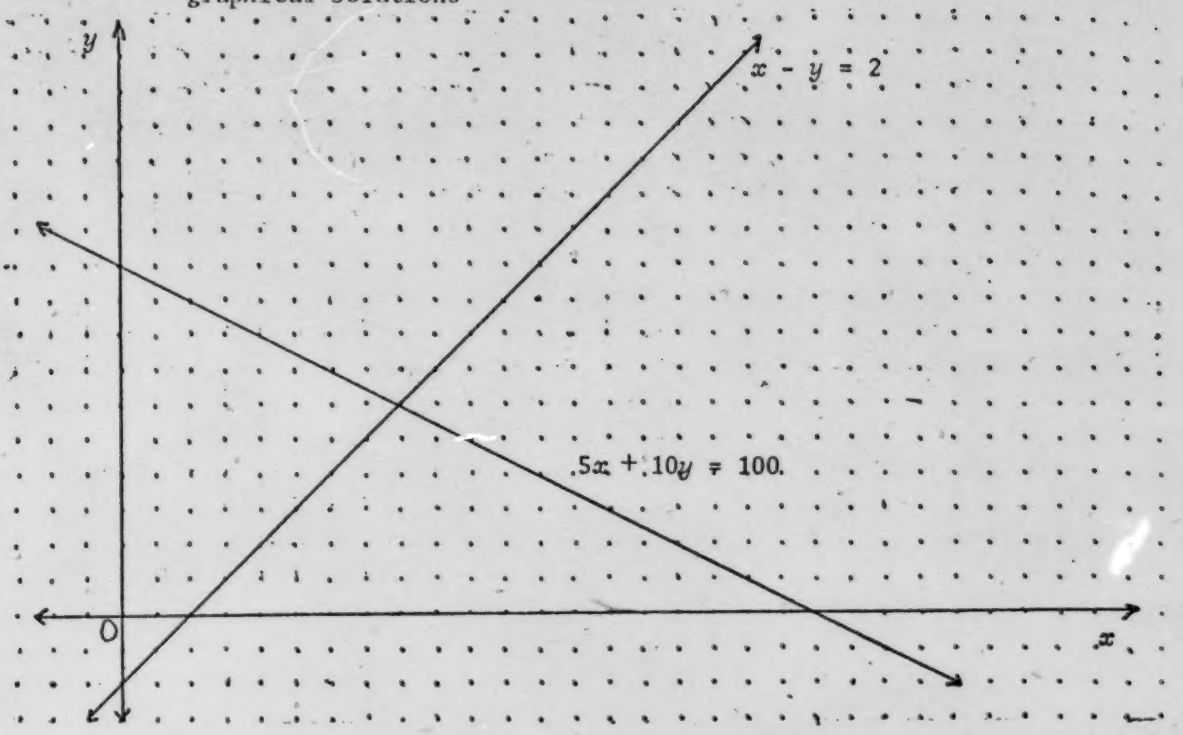
y = number of dimes

$5x + 10y = 100$

$x - y = 2$

108.2: Solution of a system of linear equations

--graphical solutions



--algebraic solutions

elimination, e.g. ① $5x + 10y = 100$

and ② $x - y = 2$

$\Rightarrow$ ③ = ① ÷ ⑤ $x + 2y = 20$

$\Rightarrow$ ④ = ③ - ② $3y = 18$

$$\Rightarrow \begin{cases} y = 6 \\ x = 8 \end{cases}$$

substitution: e.g. $x + 2y = 20$

$$x = y + 2$$

$$\Rightarrow y + 2 + 2y = 20 \quad \Rightarrow \quad 3y = 18$$

$$\Rightarrow \begin{cases} y = 6 \\ x = 8 \end{cases}$$

Cramer's Rule

$$x + 2y = 20$$

and $x - y = 2$

$$\Rightarrow x = \frac{\begin{vmatrix} 20 & 2 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}} = 8$$

$$\text{and } y = \frac{\begin{vmatrix} 1 & 20 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}} = 6$$

108.3: Inconsistent systems (graph as parallel lines)

e.g. $x + 5y = 20$

$3x + 15y = 4$

108.4: Dependent systems

e.g. graph as same line

$x + 5y = 20$

$5x + 25y = 100$

109 Finite Systems

109.1 Modular arithmetic tables

e.g.

Mod 5

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Mod 6

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Mod 5

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Mod 6

X	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

109.2: Comparative structure

e.g.--commutativity of addition and multiplication

--associativity of addition and multiplication

--additive identity (0)

--additive inverses $3 + 2 = 0 \pmod 5$

$3 + 3 = 0 \pmod 6$)

--distributivity e.g., (mod 5)

$3(2 + 1) = 3 \cdot 2 + 3 \cdot 1$

$= 1 + 3 = 4$

and $3 \cdot 3 = 4$

--multiplicative identity (1)

--multiplicative inverses

distinction between mod 5

(5 a prime) and mod 6 (6 composite)

zero divisors

e.g., $2 \cdot 3 = 0 \quad 2 \neq 0 \, , \, 3 \neq 0$

110 Finite Sets

110.1: Specifying a set

--by description

e.g., Set of factors of 60

-- by roster

{ -60, -30, -20, -15, -12, -10, -6, -5, -4,

-3, -2, -1, 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

--by property

$\{ n: \ n \mid 60 \} = \{ n: \ n \text{ is a factor of } 60 \}$

-- equality

$\{ n: n \mid 60 \}$ = the set of factors of 60

= {1, -1, 2, -2, 3, -3, 4, -4, 5, -5

6, -6, 10, -10, 12, -12, 15, -15

20, -20, 30, -30, 60, -60 }

110.2: Operations and relations on sets

--set membership

$20 \in \{n: n \mid 60\}$

--set inclusion

$\{n: n \mid 20 \} \subseteq \{n: n \mid 60\}$

--union of sets: of disjoint sets

: of non-disjoint sets

--Intersection of sets

e.g. $\{n: n \mid 12\} \cap \{n: n \mid 20\}$

$= \{1, -1, 2, -2, 4, -4\}$

--complementation and differences

e.g. Universe = U = Set of whole numbers less than 20

Complement of {0, 1, 2, 3, 4,} is the set of whole numbers less than 20 but greater than 4.

{1, 3, 5, 7} - {0, 1, 2, 3, 4,} = {5, 7}

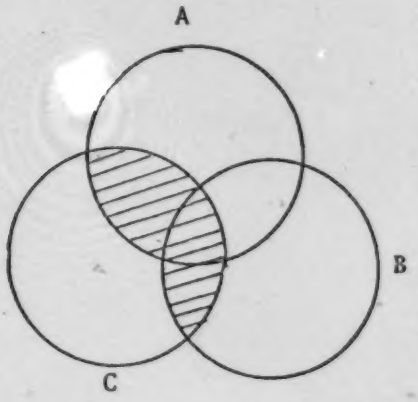
110.3: Representation of Sets.

--Venn diagrams

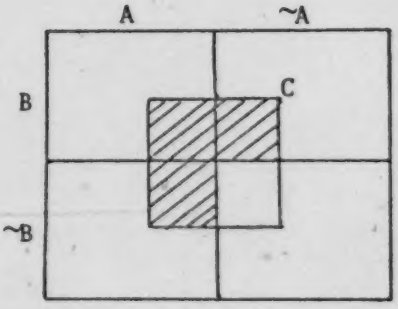
--Carrol diagrams

--Use of diagrams to represent sets

e.g.



$(A \cup B) \cap C$



$(A \cup B) \cap C$

--Use of diagrams to represent information

e.g., In a certain school with 300 students

100 study mathematics

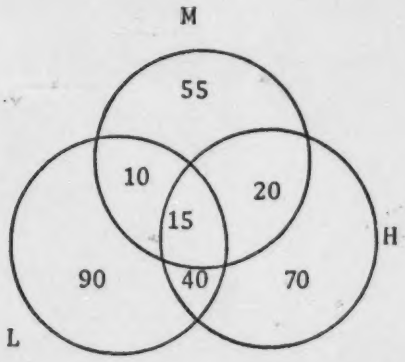
35 study mathematics and history

25 study mathematics and language

15 study mathematics, history and language

55 study history and language

155 study language



How many study history?

110.4: Properties of operations

--commutativity and associativity of union

--commutativity and associativity of intersection

--distributivity of union over intersection and of intersection over union.

111 Flowcharts and Programming

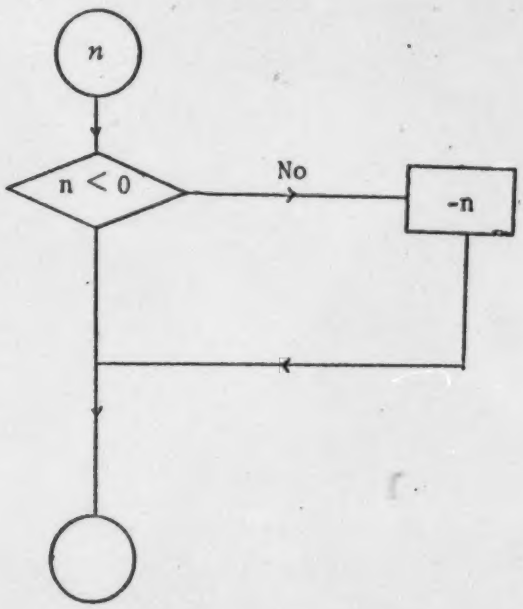
111.1 Making flowcharts

e.g., choose a number. Add 7 to 5 times your number and then double the result.

111.2: Interpret flowcharts

--reading and following flowcharts

e.g.



111.3: Making algorithms

e.g., $a, b > 0$

divide b by a, remainder r_1

if $r_1 \neq 0$ divide a by r_1, remainder r_2

if $r_2 \neq 0$ divide r_1 by r_2, remainder r_3

if $r_3 \neq 0$ divide r_2 by r_3, remainder r_4

continue until no remainder, i.e., $r_n = 0$

then r_{n-1} =gcd (a,b)

111.4: Programming

e.g.

```
10   LET   N = 1
20   LET   S = 1
30   LET   N = N + 1
40   LET   S = S + N
50   IF    N < 100  THEN  30
60 PRINT    S
70 END
```

112 Real Numbers

112.1: Decimal representation

--approximation by (finite) decimal fraction

e.g. $\sqrt{2} \doteq 1.4$

--error in approximation

$\sqrt{2} - 1.4 < .02$

$1.4 < \sqrt{2} < 1.42$

$\sqrt{2} \doteq 1.41$

$1.405 < \sqrt{2} < 1.415$

--"better" (successive) approximations

e.g. $\sqrt{2}$ 1.4

1.41

1.414

112.2: Operations with irrational numbers

e.g., $\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$ etc.

200 Geometry

201 Classification of plane figure

--classification in terms of number of edges. E.g. triangles, quadrilaterals, n-gons.

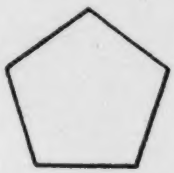
--parallel edges: trapezoids (trapeziums)

--congruent edges. E.g., two edges of one figure congruent; congruent edges of distinct figures.

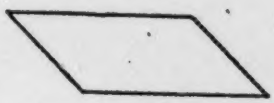
--congruent figures

--similar figures

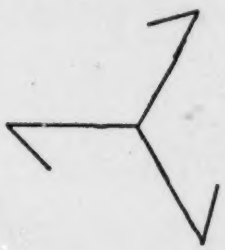
--symmetries: bilateral symmetry

e.g. 

: central (point or half-turn) symmetry

e.g. 

: rotational symmetry

e.g. 

202 Properties of plane figures

--sum of measures of interior angles

of a triangle (= 180°)

of a quadrilateral (=360°)

--parallelograms: opposite angles are congruent

: opposite sides are parallel

:opposite sides are congruent

:diagonals bisect each other

:central symmetry

--rectangles: parallelogram with interior angles right angles

:2 axes of symmetry

--squares: rectangle with all sides congruent

:4 axes of symmetry

:rotational symmetry

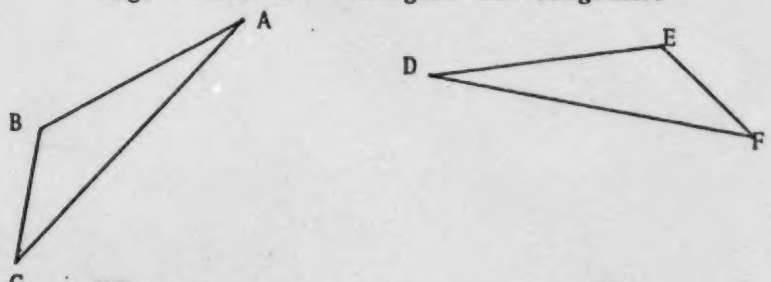
203 Congruence of plane figures

203.1 Classification-according to congruence e.g., congruent figures in one class.

--select figure from a set of figures congruent to a given one.

203.2 Properties of congruent figures-corresponding parts are congruent

e.g. These two triangles are congruent



$\overline{AB}$ is 10 cm long. How long is $\overline{DE}$?

--construction of congruent figures by tracing.

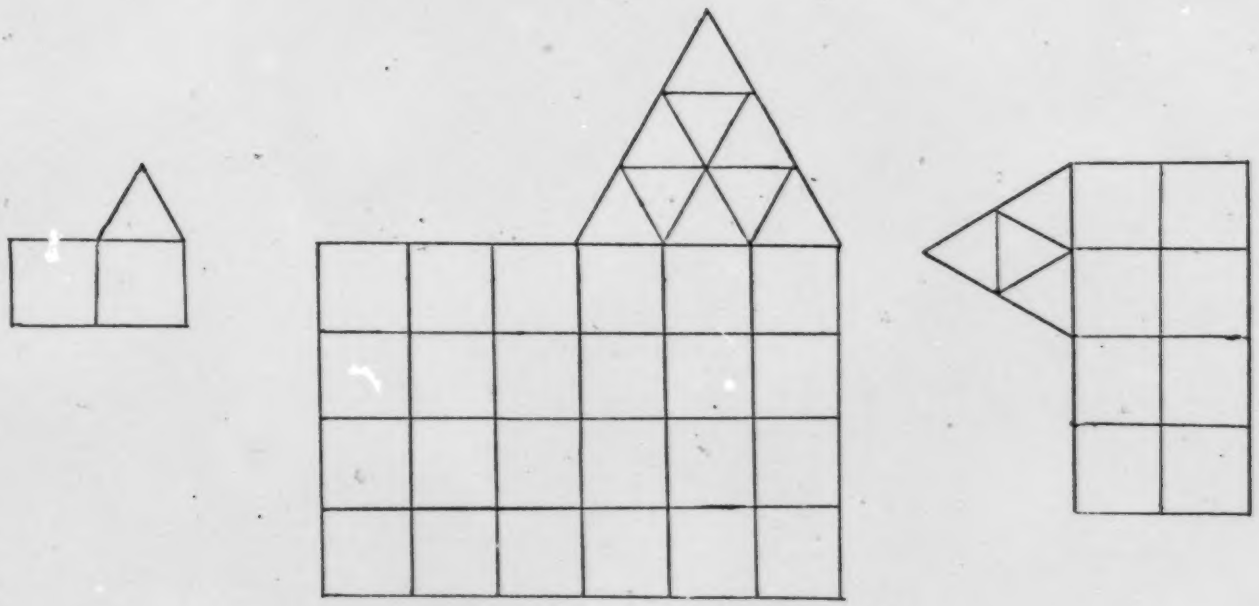
204 Similarity of Plane Figures

204.1 Classification-

--sort a collection of figures according to similarity

--select from a collection of figures one similar to a given figure

204.2 Recognition of similar figures e.g. all small triangles and all small squares are congruent. Which of these figures are similar?



204.3 Scale drawings

205 Geometric Constructions

205.1 Straight edge and compass constructions

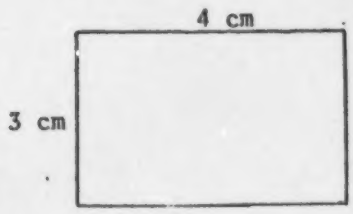
e.g. find the mid-point of a segment

205.2 Mira construction (Mirror cards)

e.g. find the mid-point of a segment.

206 Pythagorean triangles

206.1 Using Pythagoras' theorem, what is the length of a diagonal of this rectangle?



206.2 Some Pythagorean triples

3, 4, 5

5, 12, 13

206.3 Irrational numbers as lengths of hypotenuse of a right triangle

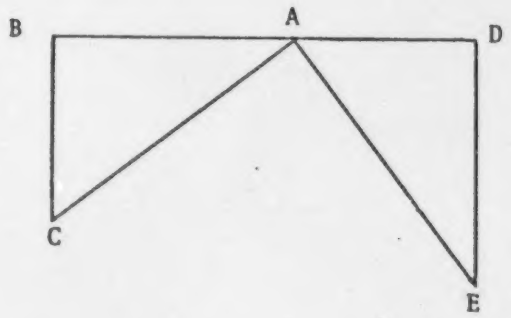
207 Coordinates

207.1 Locating points in a coordinated plane(i.e., a plane furnished with a coordinate system)

207.2 Graphing relations in a coordinated plane.

208 Simple deductions

e.g.

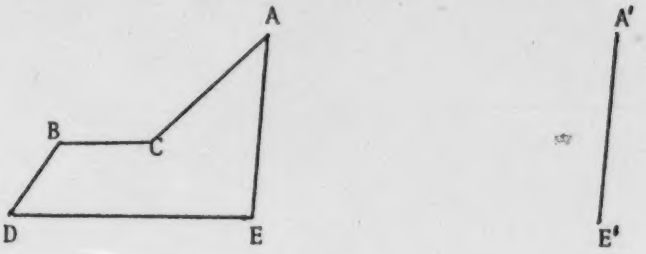


Δ ABC $\equiv$ Δ EDA, B,A,D collinear,

$\angle$ B a Rt. angle,

CAE?

209. Informal transformations in geometry.



--figure AEDBC is translated so that A → A', E → E'. Sketch the image. Figure AEDBC is reflected so that A → A', E → E'. Sketch the image.

210. Relationships between lines and planes in space.

--non-parallel planes intersect in a line

--non-parallel lines need not intersect

--skew lines

--intersecting and parallel lines are coplanar. Skew lines are not.

--line intersects non-parallel plane in a point.

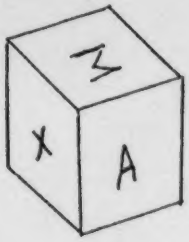
211. Solids

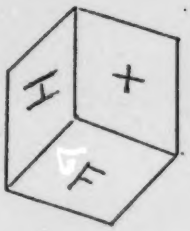
--prisms and pyramids

--symmetries: axes of symmetry

planes of symmetry

212. Spatial visualization and representation.

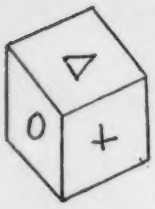


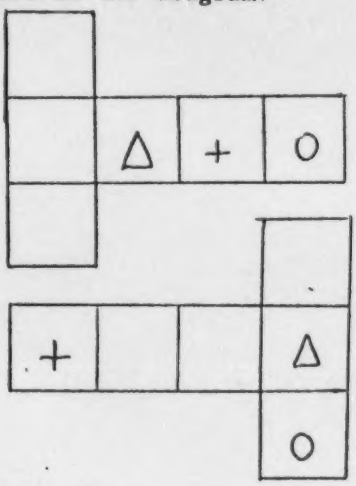


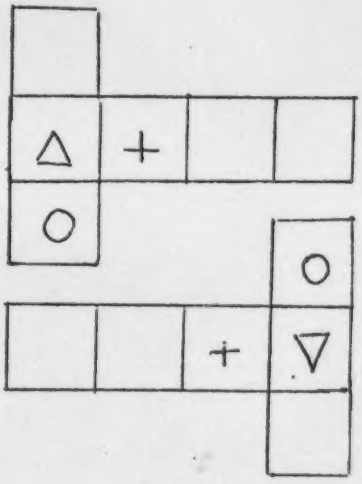
--2 views of same cube

--side not shown has 'O' on it. Which side is opposite M; O, or F?

Which nets yield the cube in the diagram?





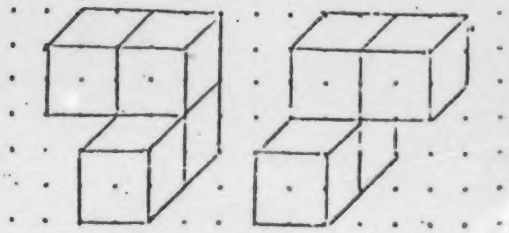


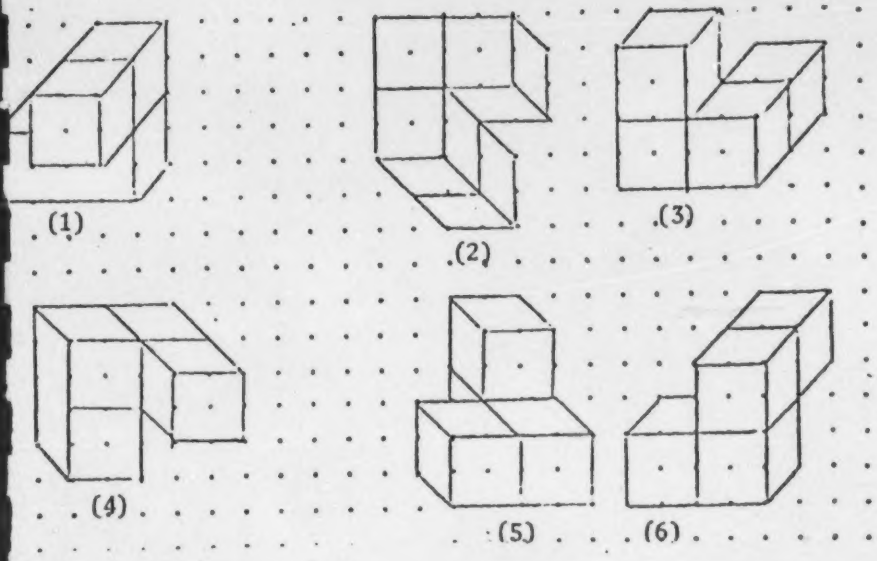
213. Orientation

213.1: --In plane: can any of these figures be rotated into an 'F'?

213.2: --In space

--which of the figures shown below can be repositioned to look like (a)? Like (b)?

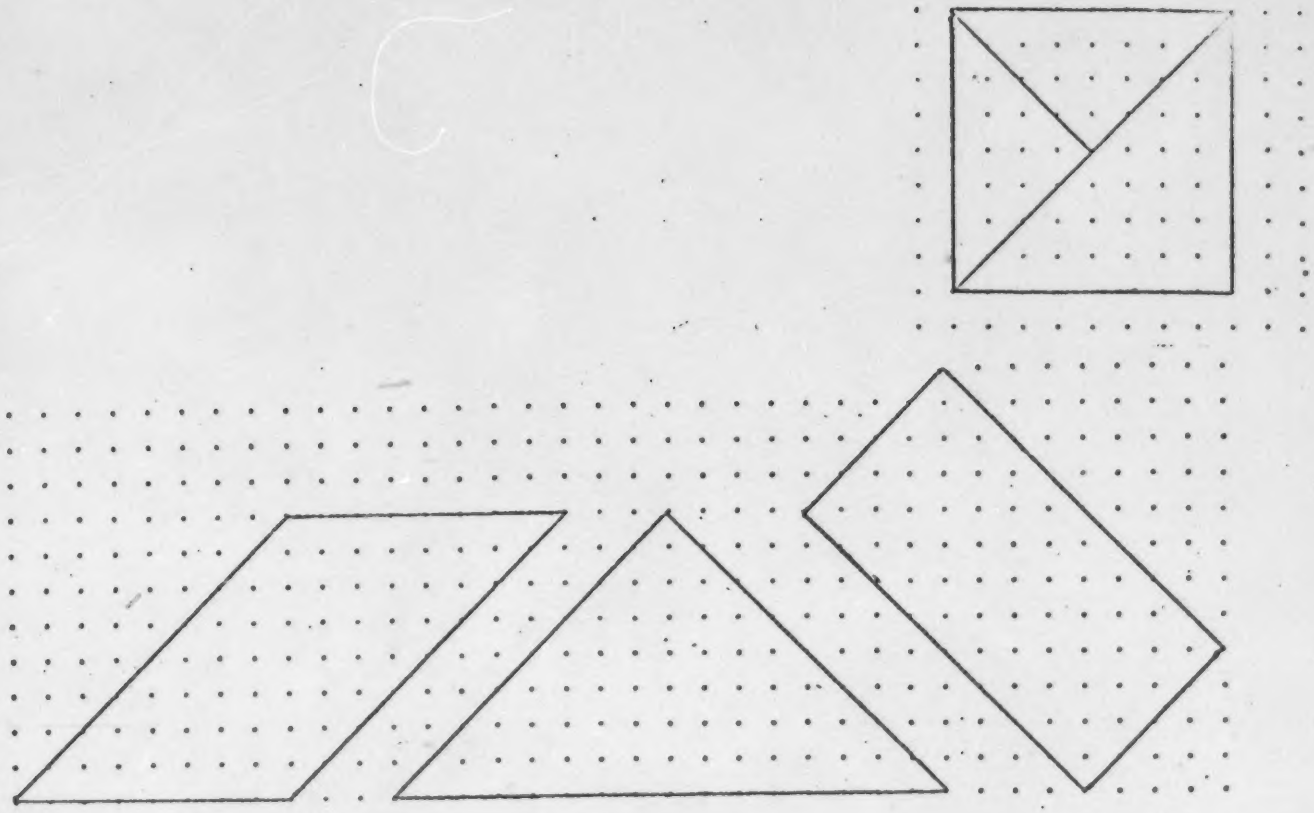




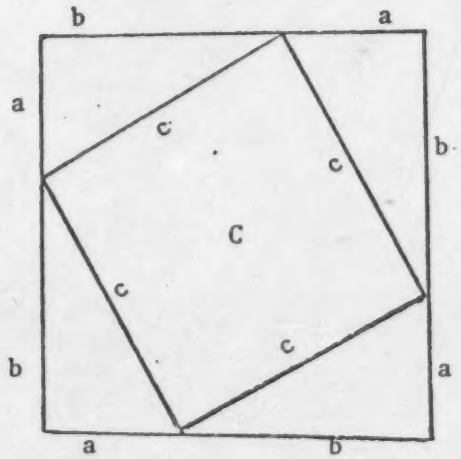
214. Decomposition of Figures

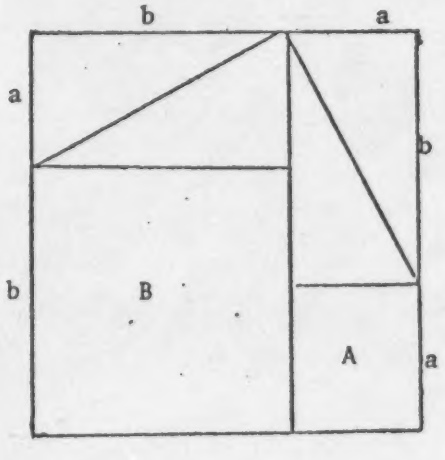
--Decomposing figures into congruent pieces:

e.g., the square on the next page has been decomposed into 3 triangles. Show how to rearrange these 3 triangles to form each of the figures below the square.



--Compare the two decompositions of the square (the length of each side is a + b). What can you conclude about region C and Regions A and B?





300 Probability and Statistics

301 Data collection - survey or census

- sampling

E.g. There are 60,000 candy coated chocolates in a box in six colours - red, green, yellow, white, orange and brown - and uniformly mixed. A sample of 200 candies taken from the box is found to contain

32	red	30	green
41	green	57	orange
26	yellow	14	brown

Estimate how many of each colour are in the box.

302 Organization of data

--tables

--frequency distributions

Data on the number of students absent from an elementary school on 80 school days:

11	20	16	10	22	7	13	14	23	11
15	4	10	12	14	18	15	13	11	6
26	16	14	18	27	29	23	33	17	24
17	18	21	11	10	12	13	18	15	16
22	14	15	8	9	17	15	14	10	11
10	13	12	17	16	20	19	10	7	12
16	18	21	19	19	15	9	13	19	18
6	11	15	20	13	24	24	17	12	25

Frequency table:

Number of Absences	Tally	Frequency
5-9	𝍸 ///	8
10-14	𝍸 𝍸 𝍸 𝍸 𝍸 ///	28
15-19	𝍸 𝍸 𝍸 𝍸 𝍸 //	27
20-24	𝍸 𝍸 //	12
25-29	////	4
30-34	/	1
	Total	80

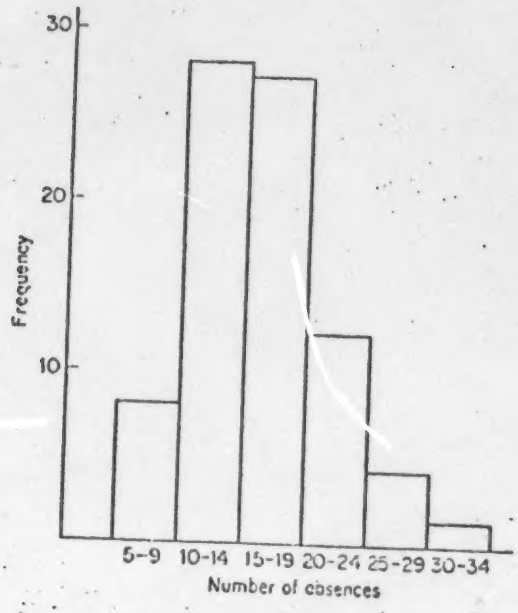
Number of Absences	Frequency	Frequency as % of Absentees.
5-9	8	10
10-14	28	35
15-19	27	33.75
20-24	12	15
25-29	4	5
30-34	1	1.25
	Total	100

-Cumulative frequency distribution

For the data on absenteeism, "less than" cumulative distribution:

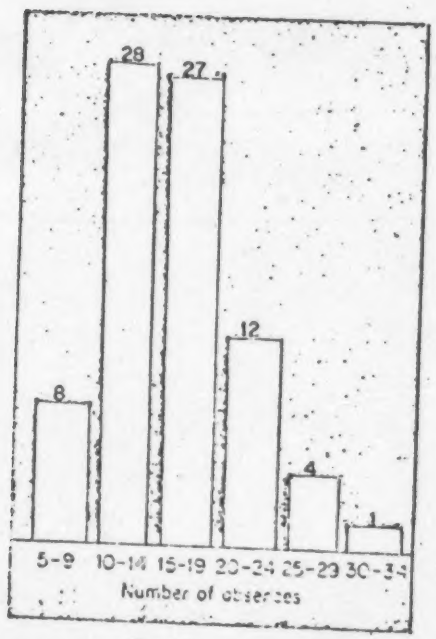
Number of Absences	Cumulative Frequency
Less than 5	0
Less than 10	8
Less than 15	36
Less than 20	63
Less than 25	75
Less than 30	79
Less than 35	80

Histogram



Histogram of distribution of absences.

Bar Graph



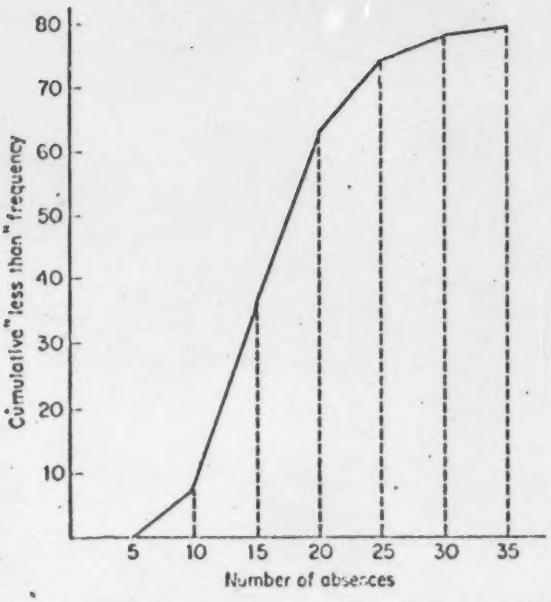
Bar chart of distribution of absences.

-Frequency polygon

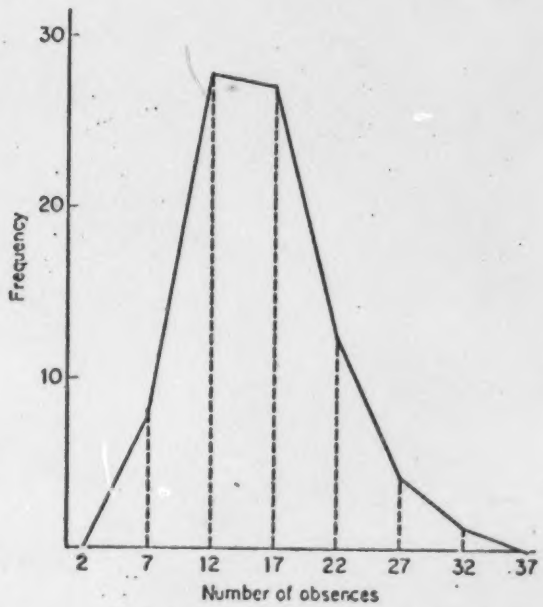
-Cumulative frequency polygon (ogive)

-Pictograph

-Pie graph or chart with graph.

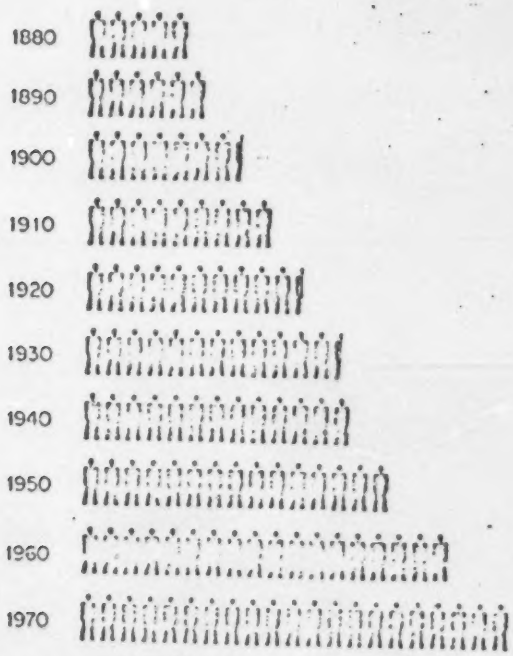


Ogive of distribution of absences.



Frequency polygon of distribution of abeences.

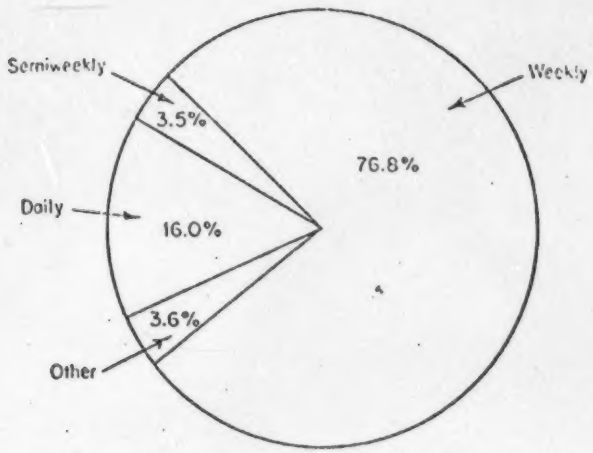
-Pictogram



Pictogram of the population of the United States.

Pie Chart

	Numbers of Newspapers
Semiweekly	398
Weekly	8,682
Daily	1,809
Other	410



Pie chart of the frequency of publication of newspapers.

304 Interpretation of data

304.1 central tendency - mean
- median
- mode

304.2 dispersion - mean deviation
- sum of squares
- variance
- standard deviation

304.3 percentiles, quartiles.

305 Combinatorics

Permutations: How many ways can we stack a red, a yellow, a blue and a green cube?

Selections: There are 3 different shaped blocks each in 4 colours. How many?

Combinations: How many ways can you form a committee of 5 from 8 people?

306 Outcomes, sample spaces and events.

e.g. Throw two dice, a red and a green, one outcome: 3 on red, 4 on green, represent by (3,4).

List outcomes - array

Red \ Green	1	2	3 . . .
1	(1,1)	(1,2)	(1,3) . . .
2	(2,1)	(2,2)	
.	.	.	
.	.	.	

Red Green

Sample space = $\{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

Event: A subspace of the sample space e.g. "Throw a nine" = {(3,6)(5,4) (4,5)(6,3)}

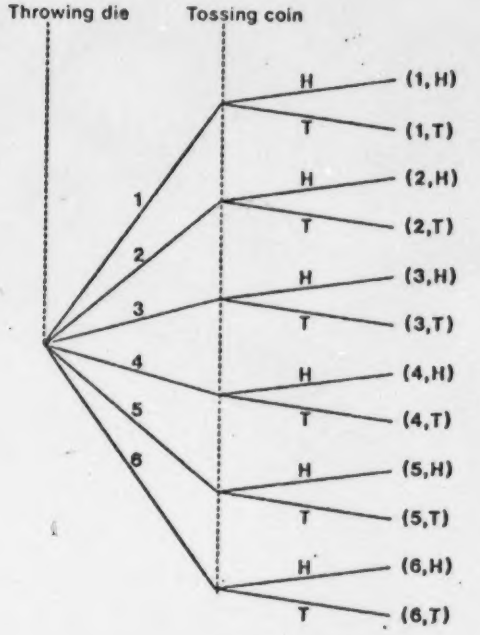
307 Counting of sets.

$N(A) + N(B) = N(A \cup B) + N(A \cap B)$

e.g. Out of a group of students, 180 take mathematics, 120 take French, 32 take both. How many students are there?

--Tree diagrams.

e.g. An experiment consists of throwing a die and tossing a coin. The following is the tree diagram illustrating the possible outcomes.



--P(AUB), P(A∩B)

e.g. $P(A) + P(B) = P(A \cup B) + P(A \cap B)$

Two dice. A = "throw a 7"

B = "one die has a 4"

$P(A) = \frac{1}{6}$ $P(B) = \frac{1}{6}$

$P(A \cap B) = \frac{1}{18}$

$P(A \cup B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{18} = \frac{5}{18}$

--Independent Events

A die is tossed twice.

A = "a three is thrown in first toss"

B = "a five is thrown on second toss"

$P(A \cap B) = P(A) \cdot P(B)$

308 Mutually exclusive events.

A = "throw a seven"

B = "throw an eleven"

$P(A \cup B) = P(A) + P(B)$

309 Complementary events

Two dice are tossed.

A ="throw a seven."

B ="do not throw a seven"

$P(A) = 1 - P(B)$

400 Measurement

401 Standard Units

401.1: --units, subunits (smaller units) and supraunits (larger units).

--e.g., metre, centimetre, kilometre.

401.2: --Relation between units.

401.3: Appropriate units

--e.g., distance from Paris to Amsterdam (kilometres)

length of a house (metres)

length of a book (centimetres)

length of a stamp (millimetres)

area of a room (square metres)

volume of a storeroom (cubic metres)

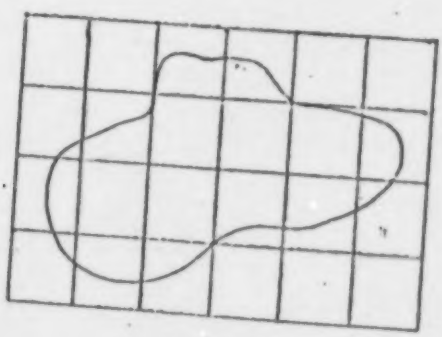
volume of gasoline (litres)

402 Estimation

--estimate length of a line segment

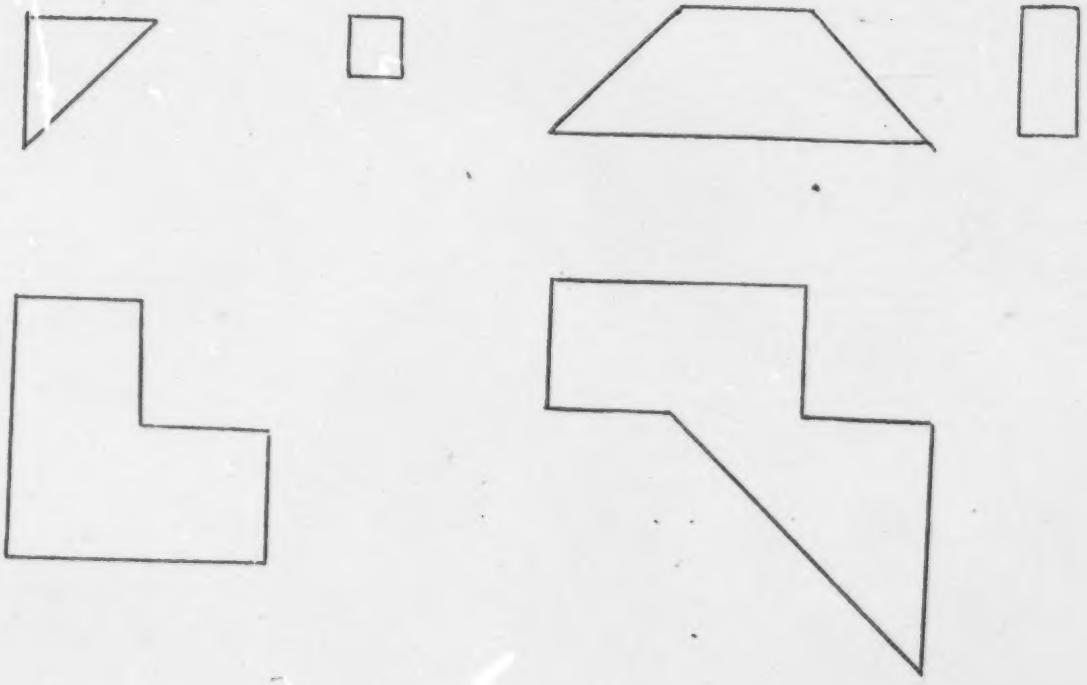
--estimate area

--e.g.,



--where this is a unit of area

Estimate the area of each of the following:



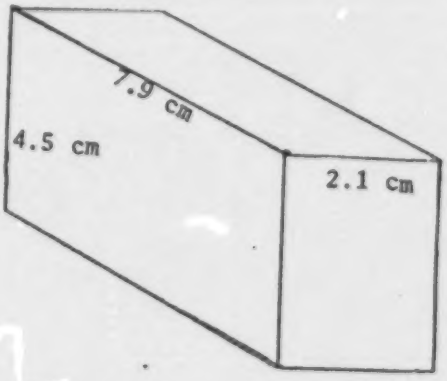
403 Approximations.

--measurements "to the nearest..."

--approximate area and volume of regular figures given dimensions

--e.g., find volume and surface area

of the given figure

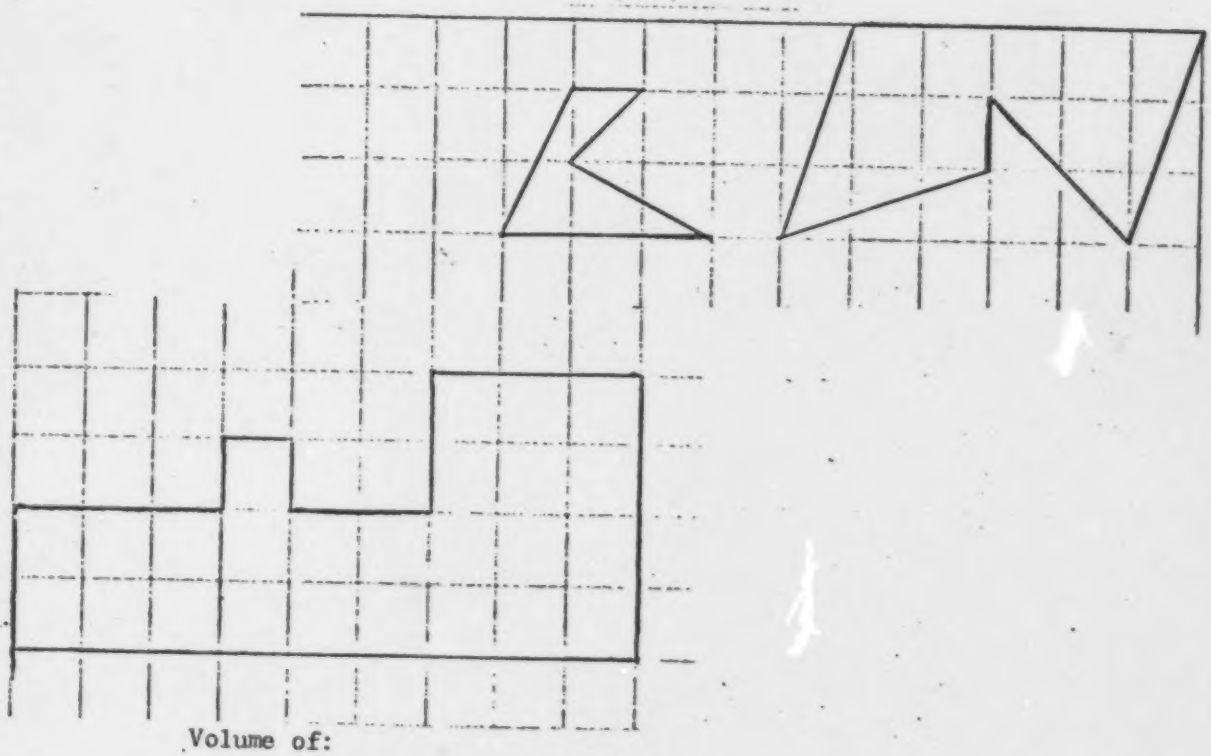


--estimation of error

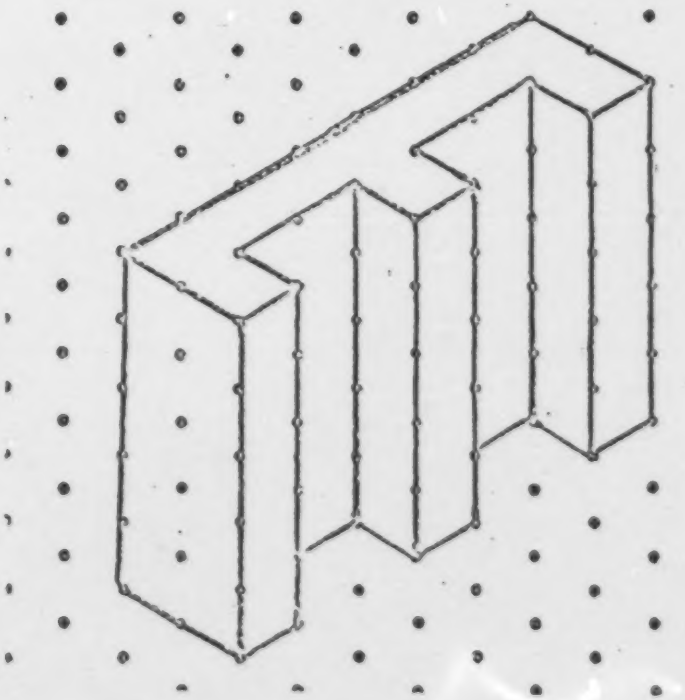
--area and volume of regular and irregular figures

--e.g., area of

CENTIMETRE GRID



Volume of:



Test Item

Test Item

Test no. A

1. 43.0 − 17.6 is equal to ——. *Ans. 25.4*

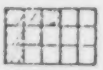
2. How many seven-man teams can you make out of 7 nine-man teams?

A. 7 B. 8 *C. 9 D. 16 E. 63

3. $(22 \times 13) - (47 + 59)$ is equal to

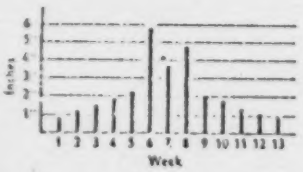
*A. 290 B. 300 C. 384 D. 408 E. 502

4. In the figure on the right the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to



A. $\frac{2}{15}$ B. $\frac{1}{3}$ *C. $\frac{2}{5}$ D. $\frac{3}{8}$ E. $\frac{1}{2}$

5. In the graph on the right, rainfall in inches is plotted for 13 weeks. The average weekly rainfall during the period is approximately



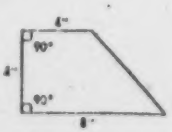
A. 1 inch D. 4 inches
*B. 2 inches E. 5 inches
C. 3 inches

6. The value of $2^3 \times 3^2$ is

A. 30 B. 36 C. 64 *D. 72 E. none of these

7. A box has a volume of 100 cc. Another box is twice as long, twice as wide and twice as high. How many cc is the volume of the second box? *Ans. 800*

8. There is a brass plate of the shape and dimensions shown in the adjoining figure. What is its area in square inches?



A. 16 *B. 21 C. 32 D. 64 E. 96

9. What is the square root of 12×75?

A. 6.25 *B. 30 C. 87 D. 625 E. 900

10. Three straight lines intersect as shown in the figure on the right. What is x equal to in degrees?

A. 30 B. 50 C. 60 *D. 110 E. 150

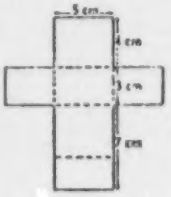
11. A shopkeeper has x lb of tea in stock. He sells 15 lb. and then receives a new lot weighing $2y$ lb. What weight of tea does he now have?

A. $x - 15 - 2y$ D. $x + 15 - 2y$
B. $x + 15 + 2y$ E. none of the above
*C. $x - 15 + 2y$

12. If $\frac{x}{2} < 7$, then

A. $x < \frac{7}{2}$ B. $x < 5$ *C. $x < 14$ D. $x > 5$ E. $x > 14$

13. A piece of tin with dimensions as shown is to be folded along the dotted lines to make a box. What is the volume, in cubic centimeters, enclosed in the box? *Ans. 60*



14. If $\frac{4x}{12} = 0$, then x is equal to

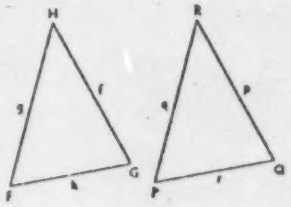
*A. 0 B. 3 C. 8 D. 12 E. 16

15. The floor of a room is covered with wooden rectangular blocks. When blocks measuring a inches by b inches are used, M blocks are needed. If blocks fit exactly, how many blocks will be needed if each block measures x inches by y inches?

*A. $\frac{Mab}{xy}$ B. $\frac{ab}{Mxy}$ C. $\frac{(a+b)M}{x+y}$ D. $\frac{ab \cdot xy}{M}$ E. $\frac{Mxy}{ab}$

Test Item

16. Which of the following sets of conditions is *not* sufficient for the congruence of $\triangle FGH$ and $\triangle PQR$ on the right when f is less than g?



*A. $\angle F = \angle P$
$g = q$
$f = p$

B. $\angle F = \angle P$
$h = r$
$\angle G = \angle Q$

C. $g = q$
$\angle F = \angle P$
$h = r$

D. $h = r$
$g = q$
$f = p$

E. $f = p$
$\angle G = \angle Q$
$h = r$

17. Which of the following is (are) true?

I. $(53 \times 73) \times 17 = 53 \times (73 \times 17)$
II. $133 \times (78 + 89) = (133 \times 78) + 89$
III. $133 \times (78 + 89) = (133 \times 78) + (133 \times 89)$

A. I only
B. II only
C. III only
D. I and II only
*E. I and III only

18. There are 227 boys in a school. Every boy in the school belongs to either the music club or the sports club, and some boys belong to both clubs. The music club has 120 members, and 36 of these are also members of the sports club. What is the total membership of the sports club? *Ans. 143*

19. The lengths of the sides of a triangle XYZ are 4, 7, and 10. If a similar triangle has a perimeter of 147, what is the length of its shortest side? *Ans. 28*

20. In the solution of the following system of equations,

$$\left.\begin{aligned} 2x + y &= 7 \\ x - 4y &= 4 \end{aligned}\right\}$$

the value of y is equal to

A. $-\frac{5}{3}$ B. -9 C. $\frac{1}{9}$ *D. $-\frac{1}{9}$ E. $\frac{5}{3}$

21. Which of the following is true for any parallelogram $ABCD$ which has an acute angle at B and diagonals AC and BD?

A. $AB < BC$
B. $AB = BC$
C. $AB > BC$
*D. $AC < BD$
E. None of them

Test Item

22. The distance between two towns, A and B, is 150 kilometers. This distance is represented on a certain map by a length of 30 centimeters. The scale of this map is

*A. 1/500,000
B. 30/150
C. 1/20,000
D. 1/5,000
E. 1/200,000

23. Which of the following equals $7 \times (3 + 9)$?

*A. $(7 \times 3) + (7 \times 9)$
B. $(7 \times 9) + (3 \times 9)$
C. $(7 \times 3) + (3 \times 9)$
D. 7×27
E. $21 + 9$

Test no. B

1. $\frac{2}{5} + \frac{3}{8}$ is equal to

A. $\frac{5}{13}$ B. $\frac{5}{40}$ C. $\frac{6}{40}$ D. $\frac{16}{15}$ *E. $\frac{31}{40}$

2. Peter and Paul decided to start saving money. Peter can save 3 dollars each month and Paul can save 5 dollars. At this rate, after how many months will Paul have exactly 10 dollars more than Peter?

A. 2 B. 3 C. 4 *D. 5 E. 8

3. In the division on the right, the correct answer is

$.01\overline{)24.56}$

A. .614
B. 6.14
C. 61.4
D. 614
*E. 6140

4. The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to

A. 2.40 *B. 2.55 C. 3.75 D. 7.65 E. none of these

5. Which of the following operations with whole numbers will *always* give a whole number?

I. Addition
II. Multiplication
III. Division

A. I only
B. II only
C. III only
*D. I and II only
E. II and III only

6. If the Selling Price of an article was $55 and a profit of 10% was made on the Cost Price, what was the cost price in dollars? *Ans. 50*

7. The value of 0.2131×0.02958 is approximately

A. 0.6 B. 0.06 *C. 0.006 D. 0.0006 E. 0.00006

Test Item

8. Joe had three test scores of 78, 76, and 74, while Mary had scores of 72, 82, and 74. How did Joe's average compare with Mary's?

A. Joe's was 1 point higher.
B. Joe's was 1 point lower.
*C. Both averages were the same.
D. Joe's was 2 points higher.
E. Joe's was 2 points lower.

9. Which of the following is *false* when a and b are different real numbers:

A. $(a+b)+c=a+(b+c)$
B. $ab=ba$
C. $a+b=b+a$
D. $(ab)c=a(bc)$
*E. $a-b=b-a$

10. If $P=LW$ and if $P=12$ and $L=3$, then W is equal to

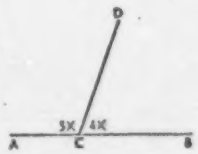
A. $\frac{3}{4}$ B. 3 *C. 4 D. 12 E. 36

11. Simplify: $5x+3y+2x-4y$

A. $7x+7y$ *D. $7x-y$
B. $8x-2y$ E. $7x+y$
C. $6xy$

12. What is the value of $(-6)-(-8)$?
Ans. $+2$

13. If AB is a straight line, what is the measure in degrees of angle BCD in the figure on the right?



A. 20 B. 40 C. 50 *D. 80 E. 100

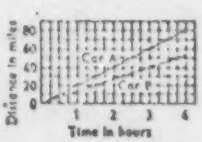
14. If $x=y=z=1$, then $\frac{x-z}{x+y}$ is equal to

A. -2 B. -1 *C. 0 D. $\frac{1}{2}$ E. 1

15. If $x=-3$, the value of $-3x$ is

A. -9 B. -6 C. -1 D. 1 *E. 9

Use the graph on the right in answering the two following questions.



Test Item

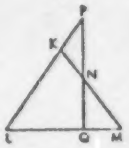
16. Three hours after starting, car A is how many miles ahead of car B?

A. 2 B. 10 C. 15 *D. 20 E. 25

17. How much longer does it take car B to go 50 miles than it does for car A to go 50 miles?

*A. 1 hour 15 minutes
B. 1 hour 30 minutes
C. 2 hours
D. 2 hours 30 minutes
E. 2 hours 35 minutes

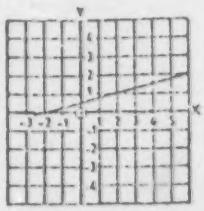
18. In $\triangle KLM$ on the right, $KL=KM$, $PO \perp LM$, and LP is a straight line. Then $\triangle NKP$ is isosceles because



*A. $\angle P=\angle KNP$, since both are complements of the equal angles L and M.
B. $NK=PK$, since $\angle P=\angle M$.
C. its sides are parallel to the sides of $\triangle KLM$.
D. its sides are perpendicular to the sides of $\triangle KLM$.
E. $\angle P=\angle KNP$, since both are half the supplement of angle M.

19. The distance between two schools on a map with a scale of 1 : 10,000, is 20 cm. What is the actual distance in kilometers between the two schools? *Ans.* 2

20. The equation of the line shown in the graph is



A. $x+4y=4$
B. $2x-y=4$
C. $2x=y-2$
*D. $x-4y+2=0$
E. $4x-y=2$

21. Which of the following numbers in base two is (are) even?

I. 110011
II. 110010
III. 110101
IV. 100100

A. I only
B. III only
C. I and III only
*D. II and IV only
E. I, III, and IV

Test Item

22. The expression $\frac{a}{b-c}+\frac{a}{c-b}$ where $a \neq 0$, is equal to

*A. 0
B. $\frac{2a}{b-c}$
C. $\frac{a}{b^2-c^2}$
D. $\frac{a}{2b}$
E. $2a$

23. Soda costs a cents for each bottle but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A. $ax+by$
*B. $ax-by$
C. $(a-b)x$
D. $(a+x)-(b+y)$
E. none of these

24. From a long stick of wood a man cut 6 short sticks each 2 feet long. He then found he had 1 foot left over. Which of the following would tell him the length of the original stick of wood?

A. $6\times(2+1)$
*B. $(6\times2)+1$
C. $(6\div2)-1$
D. $(6\times2)-1$
E. $(6\div2)+1$

Test no. C

1. Which of the following is the same as a quarter of a million?

A. 25,250
B. 40,000
C. $\frac{1}{4,000,000}$
*D. 250,000
E. 2,500,000

2. 0.40×6.38 is equal to

A. .2552 B. 2.452 *C. 2.552 D. 24.52 E. 25.52

3. The sum of $9\frac{4}{5}$ and $13\frac{1}{4}$ is equal to

A. $22\frac{5}{9}$ B. $22\frac{9}{20}$ C. 23 *D. $23\frac{1}{20}$ E. $23\frac{1}{5}$

4. The ratio of 2 to 5 equals the ratio of what number to 100? *Ans. 40*

5. In a given triangle the measures of two angles in degrees are 60 and 70. What is the measure of the third angle in degrees? *Ans. 50*

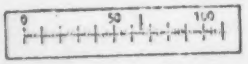
6. On level ground, a boy 5 feet tall cast a shadow 3 feet long. At the same time a nearby telephone pole 45 feet high cast a shadow the length of which, in feet, is
A. 24 *B. 27 C. 30 D. 60 E. 75

Test Item

7. A runner ran 3,000 meters in exactly 8 minutes. What was his average speed, in meters per second?

A. 3.75 *B. 6.25 C. 16.0 D. 37.5 E. 62.5

8. On the scale to the right, the reading indicated by the arrow is between



A. 51 and 52
B. 57 and 58
C. 60 and 62
D. 62 and 64
*E. 64 and 66

9. If $x+y=4$ and $x-y=2$, then x is equal to

A. 0 B. 1 C. 2 *D. 3 E. 6

10. One bell rings every 8 minutes, while another bell rings every 12 minutes. They have rung together once at the same moment. After how many minutes will they ring together again

A. for the first time? *Ans. 24*
B. for the second time? *Ans. 48*
C. for the tenth time? *Ans. 240*

11. At 4 o'clock, the measure of the angle between the minute hand and the hour hand of a clock in, degrees, is

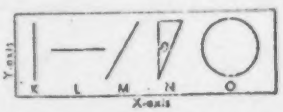
A. 30 B. 45 C. 60 D. 90 *E. 120

12. Any two regular polygons with the same number of sides are

A. congruent
B. noncongruent
*C. similar
D. not similar
E. equal in area

Questions 13–15.

Imagine that the geometrical figures K, L, M, N and O have been drawn on a rubber sheet. The lines are assumed to have no width. The rubber sheet is stretched parallel to the X-axis while leaving all the distances measured parallel to the Y-axis unchanged. The stretching is uniform, that is, the same for every part of the sheet.



13. For which of the segments K, L, M will the length remain unchanged?

*A. only K B. only L C. only M D. K and L E. K and M

Test Item

14. What will happen to the measure of angle θ of triangle N?

A. It will remain the same.
•B. It will become larger.
C. It will become smaller.
D. One cannot tell from the data whether A, B, or C is correct.

15. What will happen to circle O?

A. It will still be a circle.
•B. It will no longer be a circle.
C. One cannot tell from the data whether A or B is correct.

16. A factory produces m units per week. How many units per week will it produce after production is increased p percent?

A. $100p + m$ C. $\frac{m + mp}{100}$ E. $\frac{p}{100} + m$

B. $100m + mp$ •D. $m + \frac{mp}{100}$

17. Let the symbol $\overline{a, b}$ denote the set of integers between a and b. For example, $\overline{3,7}$ cosists of the integers 4, 5, and 6. Which of the following pairs of sets has a larger number of integers in common than any of the other pairs?

A. $\overline{0,15}$ and $\overline{7,20}$ •D. $\overline{4,18}$ and $\overline{8,20}$
B. $\overline{5,15}$ and $\overline{16,30}$
C. $\overline{5,14}$ and $\overline{5,17}$ E. $\overline{0,12}$ and $\overline{6,12}$

18. What are all values of x for which the inequality

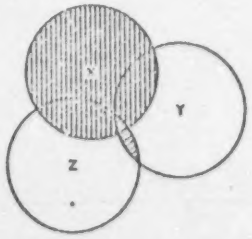
$$5x + \frac{5}{3} \leq -2x - \frac{2}{3}$$

is true?

A. $x \leq -\frac{7}{9}$ C. $x \geq 0$ E. $x \geq \frac{9}{3}$

•B. $x \leq -\frac{1}{3}$ D. $x \geq \frac{7}{3}$

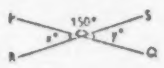
19. The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q. Which of the following represents the shaded portion of the diagram below?



A. $(X \cap Y) \cup Z$ C. $X \cap (Y \cup Z)$ E. $(X \cup Y) \cap Z$
•B. $X \cup (Y \cap Z)$ D. $(X \cap Y) \cap Z$

Test Item

20. If, in the given figure, PQ and RS are intersecting straight lines, then $x + y$ is equal to



A. 15 B. 30 •C. 60 D. 180 E. 300

21. Each of 9 boys had t marbles. In order to play a game, they divided the marbles among 12 boys in such a way that each had the same number. How many marbles did each of the 12 have?

•A. $\frac{3t}{4}$ B. $t - 3$ C. $\frac{4t}{3}$ D. $9t - 12$ E. $12t - 9$

22. The length of the circumference of the circle on the right with center at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?

A. 24 B. 30 C. 45 •D. 60 E. 90

23. Given any fraction whose numerator is less than the denominator, if you then add 2 to both the numerator and the denominator, the new fraction is

A. equal to the original fraction
•B. larger than the original fraction
C. twice the original fraction
D. smaller than the original fraction
E. 1 more than the original fraction

Test no. 5

1. If $a = 20$, $b = 0$, $c = 10$, $x = 8$, $y = 12$, then the value of $2aby + 2cx$ is

A. 100 •B. 160 C. 400 D. 640 E. none of these

2–5. For each of the following equations or pairs of equations, concerned with real numbers, mark on the answer sheet

A. if there is *no* solution
B. if there is *one* solution
C. if there are *two* solutions
D. if there are *three* solutions
E. if there are *more than three* solutions

2. $x + y = 12$, $x - y = 4$
Ans. B

3. $m + n = 2$, $3m + 3n = 9$
Ans. A

4. $x^2 - 5x + 6 = 0$
Ans. C

Test Item

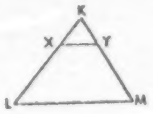
5. $3p + q = 16$

Ans. E

6. If $xy = 1$ and x is greater than 0, which of the following statements is true?

A. When x is greater than 1, y is negative.
B. When x is greater than 1, y is greater than 1.
C. When x is less than one, y is less than 1.
D. As x increases, y increases.
*E. As x increases, y decreases.

7. In the figure on the right, $KX = \frac{1}{3} KL$ and $KY = \frac{1}{3} KM$. Which of the following statements are true?



I. $XY = \frac{1}{3} LM$

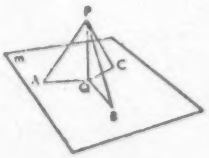
II. Line XY is parallel to line LM

III. Area $KXY = \frac{1}{3}$ area KLM

IV. Area $KXY = \frac{1}{9}$ area KLM

A. I and II only
B. II and III only
C. I and III only
D. I, II and III only
*E. I, II, and IV only

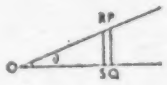
8. In the figure on the right, m represents a plane, and PQ is a line segment which is perpendicular to the plane at the point Q. Points A, B and C lie on the plane. If $QA = QB = QC$, then the triangles PQA, PQB, and PQC are



*A. congruent by *SAS*
B. congruent by *SSA*
C. congruent by *ASA*
D. similar but not congruent
E. neither similar nor congruent

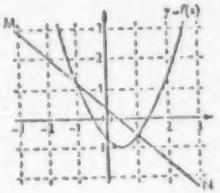
Test Item

9. In the figure below, $PQ \perp OQ$, and $RS \perp OQ$. If the measure of OQ and of OR equal 1 and θ is the measure of $\angle POQ$, then the measure of segment PQ is equal to



A. $\sin \theta$
B. $\cos \theta$
*C. $\tan \theta$
D. $2 \sin \theta$
E. $1 - \cos \theta$

Questions 10 and 11 are based upon the graph of a quadratic equation which is shown in the figure on the right



10. For what value of x is the quadratic function a minimum?

A. -1 B. $-1/2$ *C. $1/2$ D. 1 E. $1\frac{1}{2}$

11. The range of values of x for which the function represented by the straight line MN exceeds the quadratic function is

*A. $-1 < x < 1$
B. $x < -1$ and $x > 1$
C. $-3/4 < x < 1\frac{1}{4}$
D. $x > 0$
E. $x > y$

12. A square plate of the largest possible size is cut from a circular plate of 16 cm diameter. The area of the square plate (in sq cm) will be

A. 64 B. 96 *C. 128 D. 192 E. 256

13. The locus of all mid points of chords drawn from a point of a circle is

A. a semicircle
*B. a circle
C. a straight line
D. an oblong
E. none of these

14. A piece of wire 52 inches long is cut into two parts and each part is bent to form a square. The total area of the two squares is 97 square inches. What is the length in inches of the side of the smaller square? *Ans. 4*

15. The complex number $(1 + i)^2$ is equal to

A. 0 B. 2 *C. $2i$ D. $1 + i$ E. $2 + 2i$

Test Item

16. Given $\log_5 2 = \frac{1}{3}$, $\log_5 32$ is equal to

A. 2 B. 5 C. $-\frac{3}{5}$ *D. $\frac{5}{3}$ E. $\frac{3}{\log_5 32}$

17. Below there are several definitions of new operations named * in terms of the usual operations on real numbers. For which of the definitions is the property $y * x = x * y$ valid for all positive real numbers x and y?

A. $x * y = \frac{x}{y}$ *D. $x * y = \frac{xy}{x+y}$

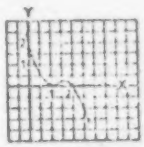
B. $x * y = x - y$

C. $x * y = x(x+y)$ E. $x * y = x^2 + xy^3 + y^4$

18. Solve the equation

$\sqrt{x+5} - \sqrt{x-3} = \sqrt{x}$ *Ans. 4*

19. The graph on the right is the representation of one of the following equations. Which one does it represent?



A. $y = (1-x)(x-2)$

B. $y = (1-x)(2-x)$ D. $y = (1-x)^2(x-2)$

C. $y = (1-x)(2-x)^2$ *E. $y = (1-x)^2(2-x)$

20. The expression

$$\frac{2}{\sqrt{5}} + \frac{\sqrt{45}}{5} + \frac{1}{\sqrt{5}-2}$$

is equal to

*A. $2\sqrt{5}+2$ D. $2\sqrt{5}$

B. $2\sqrt{5}-2$ E. $2-2\sqrt{5}$

C. 2

21. Chords of the same length are drawn in two circles of unequal radii. Which of the following is true?

*A. The chord in the larger circle could be equal to the radius of the smaller circle.

B. The chord in the smaller circle could not be a diameter.

C. The distance from the center to the chord is less in the larger circle.

D. The minor arc intercepted on the larger circle is longer.

E. The minor arc intercepted on the larger circle subtends the greater angle at the center.

Test no. 6

1. On the outside of two sides of a rectangular plot 25 by 20 meters there is a path of uniform width. The area covered by the path equals one half the area of the plot. What is the width of the path in meters? *Ans. 5*

Test Item

2. According to one plan for travelling to Mars, the round trip would take almost exactly three years, including a stay on Mars of 449 earth days. If one must go 34,000,000 miles each way on the trip, which of the following can be used to give an estimate of the average speed of travel in miles per hour?

A. $\frac{(3 \times 365 - 449) \times 24}{34{,}000{,}000}$ D. $\frac{34{,}000{,}000 \times 24}{2 \times (3 \times 365 - 449)}$

B. $\frac{(3 \times 365 - 449) \times 24}{34{,}000{,}000 \times 2}$ *E. $\frac{2 \times 34{,}000{,}000}{(3 \times 365 - 449) \times 24}$

C. $\frac{34{,}000{,}000}{(3 \times 365 - 449) \times 24}$

3. Of three wires, each 36 in. long, one is bent into a square, another into a rectangle with length and width in the ratio of 2:1 and the third into an equilateral triangle. Which one of the following statements describes the correct relationship between the enclosed areas?

*A. The area of the square is the greatest and that of the triangle is the least.

B. The area of the rectangle is the greatest and that of the triangle is the least.

C. The area of the triangle is the greatest and that of the square is the least.

D. The area of the triangle is the greatest and that of the rectangle is the least.

E. The areas of the square and the rectangle are the same, but the area of the triangle is less than that of the square or the rectangle.

4. If $3^{x+y} = 81$ and $25^{x/y} = 5$, then y is

A. 0 B. 2 *C. 3 D. $\frac{7}{2}$ E. $\frac{15}{4}$

5. A certain number of students are to be accommodated in a hostel. If 2 students share each room, then 2 students will be left without any room. If 3 students share each room, then 2 rooms will be left unoccupied. How many rooms are there in the hostel? *Ans. 8*

6. Four persons whose names begin with different letters are placed in a row, side by side. What is the probability that they will be placed in alphabetical order from left to right?

A. 1/120 *B. 1/24 C. 1/12 D. 1/6 E. 1/4

7. A number is the multiplicative inverse of another number if the product of the two numbers is 1. Which of the following sets of numbers is identical to the set of its multiplicative inverses?

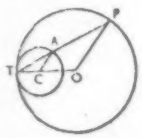
A. {1, 2, 3} D. {2, 3, 5, $\frac{1}{2}$, $\frac{1}{3}$}

B. {1, $\frac{1}{2}$} E. {2, 3, $\frac{1}{3}$}

*C. {1, 2, $\frac{1}{2}$}

Test Item

8. In the figure to the right the circle with center C is internally tangent at point T to the circle with center O. P is a point on the larger circle such that TP is not a diameter. If TP intersects the smaller circle at A, then what additional information is needed to prove that AC and PO are parallel?



•A. None
B. $PO = 2AC$
C. $TA = AP$
D. $CO = 2TC$
E. $TA = AC$

9. Which of the following is (are) true for all values of θ for which the functions are defined?

I. $\sin(-\theta) = -\sin\theta$
II. $\cos(-\theta) = -\cos\theta$
III. $\tan(-\theta) = -\tan\theta$

A. I only
B. II only
C. III only
•D. I and III only
E. II and III only

10. A radio-active element decomposes according to the formula

$$y = y_0 \cdot e^{-kt}$$

where y is the mass of the element remaining after t days and y_0 is the value of y for $t = 0$. Find the value of the constant k for an element whose half-life (i.e., time to decompose half of the material) is 4 days.

•A. $\frac{1}{4}\log_e 2$
B. $\log_e \frac{1}{2}$
C. $\log_4 e$
D. $(\log_e 2)^{1/4}$
E. $2e^4$

11. A stationer wants to make a card 8 cm long and of such a width that when it is cut into halves, the original width becomes the length and the shape of each half is similar to the original card. What width, in cm, should he make the original card?

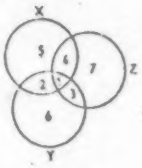
A. 4 •B. $4\sqrt{2}$ C. $5\sqrt{2}$ D. $5\sqrt{3}$ E. 6

12. The arithmetic mean or average of one group of 100 pupils is exactly 80 and the mean of another group of 50 pupils is exactly 65. What is the mean of the combined group of 150 pupils?

A. 79 B. 72.5 •C. 75 D. 77.5
E. It is impossible to determine exactly.

Test Item

13. In the diagram on the right, the numbers represent regions.
The circle X represents the set of regular polygons
The circle Y represents the set of quadrilaterals
The circle Z represents the set of equilateral triangles
Which are the parts of the schema that are empty (have no elements)?



A. 1, 3 and 5
B. 2, 3, and 4
C. 1, 6, and 7
•D. 1, 3, and 7
E. 3, 6, and 7

14. In the figure on the right, $FGHJ$ is a parallelogram. Which of the following statements is a condition which implies that $FGHJ$ is a rectangle?

A. $JF = GH$
B. $\angle HJG = \angle JGF$
•C. $\angle HJF = \angle JHG$
D. $\angle HJF$ and $\angle JGH$ are supplementary
E. HF and JG are perpendicular bisectors of each other

15. What is the sum of the infinite geometric series

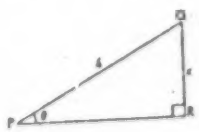
$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots ?$$

A. $\frac{5}{8}$ •B. $\frac{2}{3}$ C. $\frac{3}{5}$ D. $\frac{3}{2}$ E. ∞

16. A freight train traveling at 50 miles per hour leaves a station 3 hours before an express train which travels in the same direction at 90 miles an hour. How many hours will it take the express train to overtake the freight train?

A. $\frac{5}{9}$ B. $\frac{9}{5}$ C. $\frac{12}{5}$ •D. $\frac{15}{4}$ E. $\frac{18}{4}$

17. In right triangle PQR (at the right) the measure of PQ is 4 and θ can be any angle between 30° and 45°. What are all possible values for x, the length of RQ?



A. $0 < x < 4$
B. $\frac{1}{2} < x < \frac{\sqrt{2}}{2}$
C. $\frac{1}{2} < x < \frac{\sqrt{3}}{2}$
•D. $2 < x < 2\sqrt{2}$
E. $2 < x < 2\sqrt{3}$

Test Item

Test no. 7

1. The expression $|x-1|=1$ implies that

A. x is between 0 and 2
*B. x is either 0 or 2
C. x is less than 2
D. x is 0
E. x is 2

2. When $(1+p)^6$ is expanded, the coefficient of p^4 is

A. 6 B. 10 *C. 15 D. 20 E. 30

3. What is the converse of the statement, "If two angles are vertical, then they are equal"?

A. If two angles are vertical, then they are not equal.
*B. If two angles are equal, then they are vertical.
C. If $\angle x$ and angle $\angle y$ are vertical angles, then $\angle x = \angle y$.
D. If two angles are not vertical, then they are not equal.
E. If two angles are not equal, then they are not vertical.

4. Suppose you have proved the two theorems:

I. If p then q.
II. If s then not q.

Which of the following theorems is implied by theorems I and II?

A. If p then s.
B. If not p then not q.
C. If p or q then s.
*D. If s then not p.
E. If not s then q.

5. A train travelled a certain distance at a constant speed. Had the speed been 8 mph greater, the trip would have taken one hour less. Had the speed been 12 mph less the trip would have taken two hours more. How many miles did the train go? *Ans. 720*

6. A wholesale merchant bought a television set at a certain price and then sold it to a retail merchant at an increase of P per cent of this price. The retail merchant sold the set to a consumer for P per cent more than he paid for it. If the customer paid 65 per cent more than the price originally paid by the wholesale merchant, then P satisfies the equation:

A. $1+\frac{2P}{100}=1.65$
*B. $\left(1+\frac{P}{100}\right)^2=1.65$
C. $1+\left(\frac{P}{100}\right)^2=1.65$
D. $1+P^2=1.65$
E. $1+2P=1.65$

Test Item

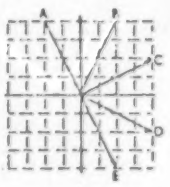
7. If a relation R is such that xRy and yRz implies xRz for each x, y, and z of a given set, the relation R is said to be transitive on that set. Which of the following relations are transitive?

I. "is father of"
II. "is contemporary of"
III. "is admirer of"
IV. "is multiple of"
V. "is perpendicular to"

A. II, IV, and V
B. I and II
C. II, III, and IV
*D. II and IV
E. V only

8. In the figure shown to the right, which vector is a graphical representation of the complex number $4-2i$?

*Ans. *D*



9. Solve $0<x^2-3x+3<7$ *Ans.* $-1<x<+1$ or $2<x<4$

10. A relation R from a set S to a set T is a function if and only if given an $x \in S$ there exists at most one $y \in T$ such that xRy.

Which of the following relations are functions?

I. x divides y
II. x has y for mother
III. x is parallel to y
IV. x has y for double
V. x has y as majorant (i.e., $x<y$)
VI. $x^2=y$

A. I, II, and III
B. II, IV, and V
*C. II, IV, and VI
D. IV, V, and VI
E. I, IV, and V

11. What is the equation whose roots are the squares of the roots of

$$x^2-5x+3=0?$$

*A. $x^2-19x+9=0$
B. $x^2+19x+9=0$
C. $x^2-20x+9=0$
D. $x^2+19x-9=0$
E. $x^2-9x+19=0$

Questions 12 and 13. Six operations are defined as follows:

$$A=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} \quad B=\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix} \quad C=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$$

$$D=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} \quad E=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} \quad F=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$$

Test Item

The operation $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, for example, means that the numbers in the upper row are transformed into the digits in the lower row, so that 1→2 (1 becomes 2), 2→3 (2 becomes 3), and 3→1 (3 becomes 1). $A \cdot B$ shows that operation B is to be performed after operation A; that is, according to A, 1→2, 2→3, 3→1, and then, according to B, 2→1, 3→2, 1→3. Therefore $A \cdot B$ will be 1→2→1, 2→3→2, and 3→1→3. This produces the same outcome as

$F = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$; let us write this $A \cdot B = F$.

In like manner, $A \cdot C$ is 1→2→3, 2→3→2, 3→1→1, and is the same as D; that is to say, $A \cdot C = D$.

12. Which one operation is equal to $C \cdot D$? *Ans. B*

13. What operation must be performed after operation B so that the combined operations are to be the same as operation F? *Ans. A*

14. If x and y belong to the set of real numbers and sets P, Q, and R are defined as follows,

$P = \{(x, y) \mid x^2 + y^2 = 4\}$
$Q = \{(x, y) \mid x - y = 2\}$
$R = \{(x, y) \mid (x^2 + y^2 - 4)(x - y - 2) = 0\}$,

which of the following is true?

A. $R = P \cap Q$ D. $R = \{\}$ (the empty set)
*B. $R = P \cup Q$ E. $R = \{(2,0)(0,-2)\}$
C. $R = \{(2,0)(0,2)(-2,0)(0,-2)\}$

15. The value $\begin{vmatrix} 4 & 2 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$ is

*A. −2 B. 0 C. 2 D. 7 E. 9

16. Each root of $x^2 - 2x - 5 = 0$ differs from the cube of the other by a positive constant c. What is the value of c? *Ans. 12*

17. Two of the roots of the equation $x^4 - 27x^3 - 14x + 120 = 0$ are 2 and 5. Find the two other roots of the equation. *Ans. −3, −4*

Test no. 8

1. If $\log_a 8 = \frac{3}{2}$, what is the value of a?

A. 2/3 B. 2 *C. 4 D. 5 E. 6

Test Item

2. If x and y are real numbers, for which x can you define y by

$$y = \frac{x}{\sqrt{9 - x^2}}?$$

A. All x except $x = 3$ *D. $-3 < x < 3$
B. All x except $x = 3$ and $x = -3$ E. $x < 3$
C. $x < -3$ and $x > 3$

3. A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A. 1/6 B. 5/24 C. 1/4 *D. 1/3 E. 5/12

4. An angle θ is known to be between 180° and 270° and $\cos^2 \theta = 16/25$. The value of $\sin 2\theta$ is then

A. $-\frac{24}{25}$ B. $-\frac{15}{25}$ C. $-\frac{7}{25}$ D. $\frac{7}{25}$ *E. $\frac{24}{25}$

5. For some functions the relationship holds that $f(x + y) = f(x) + f(y)$ for all numbers x and y. For example, when $f(x) = 2x$, then $f(x + y) = 2(x + y) = 2x + 2y = f(x) + f(y)$, i.e., $f(x + y) = f(x) + f(y)$. We call such functions as these additive. Which of the following functions is additive by this definition?

A. $f(x) = x^2$ D. $f(x) = 2^x$
B. $f(x) = \sin x$ *E. None of them are additive
C. $f(x) = \log_{10} x$

6. If determinants are used to solve the system of equations $\begin{cases} 2x + y = 3 \\ x + 4y = 7, \end{cases}$ then y is equal to

A. $\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} \Big/ \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix}$ *C. $\begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix} \Big/ \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}$ E. $\begin{vmatrix} 3 & 7 \\ 2 & 1 \end{vmatrix} \Big/ \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}$

B. $\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} \Big/ \begin{vmatrix} 3 & 1 \\ 7 & 4 \end{vmatrix}$ D. $\begin{vmatrix} 3 & 1 \\ 7 & 4 \end{vmatrix} \Big/ \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}$

Items 7 to 9 refer to the information below.

Consider the following abstract mathematical system:

Undefined terms: elements $a, b, c \ldots$ of class C; operations λ and $*$ relation $=$, having the conventional meaning of "equals".

Postulates: If a, b, and c are any elements of C, then

(1) $a \lambda b$ and $a * b$ are elements of C.
(2) $a \lambda b = b \lambda a$.
(3) $a * (b * c) = (a * b) * c$.
(4) $a * b \neq b * a$, provided $a \neq b$.
(5) $a \lambda (b * c) = (a \lambda b) * (a \lambda c)$.

Test Item

DIRECTIONS: Answer each item, using the code

A – if the proposition follows logically from the postulates.
B – if the proposition is inconsistent with the postulates (i.e., contradicts the postulates).
C – neither *A* nor *B* (i.e., the proposition neither follows from the postulates nor is contradicted by them).

7. $(a*b)\lambda c = (c\lambda a)*(c\lambda b)$ *Ans. A*

8. $(a*b)\lambda c = (a\lambda c)*(b\lambda c)$ *Ans. A*

9. $a\lambda(b*c) = (a\lambda c)*(a\lambda b)$ *Ans. B*

10. The graph of $y = f(x)$ is a parabola with axis parallel to the Y-axis. If the maximum value of y is 2, and if the parabola crosses the X-axis at $x = -\frac{1}{2}$ and at $x = \frac{3}{2}$, then its equation is

•A. $y = -2x^2 + 2x + \frac{3}{4}$
B. $y = -4x^2 - 4x + 3$
C. $y = -4x^2 + 4x + 3$
D. $y = 4x^2 - 4x - 3$
E. $y = 4x^2 + 4x - 3$

11. For what values of the real number x is $y = \frac{1}{x}$ a decreasing function?

A. No x B. $x < 0$ •C. $x \neq 0$ D. $x > 0$ E. All x

12. Solve: $2 \cdot 7^{3x+2} + 3 \cdot 7^{3x+2} = 161$
Ans. $x = -1$

13. Given two arbitrary sets X and Y. Which of the following sets is equivalent to the set

$$(X \cup Y) \cap (X \cap Y)?$$

A. X
B. Y
C. $X \cup Y$
•D. $X \cap Y$
E. $(X \cup Y) \cup (X \cap Y)$

14. Consider the matrices

$$A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 \\ y & 1 \end{pmatrix}$$

where x and y are real numbers and $x^2 + y^2 \neq 0$. For which values of x and of y is the product of the matrices commutative?

I. $x = 0$
II. $y = 0$
III. $x = y$

A. Only I
B. Only II
C. Only III
•D. Both I and II
E. I, II, and III

Test Item

15. Calculate $\arcsin \frac{1}{2} + \arcsin \frac{1}{\sqrt{2}}$ [Arc sin means "angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is"]

•A. $\frac{5\pi}{12}$
B. $\frac{7\pi}{18}$
C. $\frac{\pi}{3}$
D. $\arcsin\left(\frac{1+\sqrt{2}}{2}\right)$
E. $\arcsin \frac{\sqrt{3}}{2}$

16. For what values of x is the function

$$\frac{(1-x)(1+3x)}{(2x-1)(x-2)}$$

positive? *Ans.* $-\frac{1}{3} < x < \frac{1}{2}$ and $1 < x < 2$

Test no. 9

1. In a Cartesian coordinate system, what is the equation of the straight line passing through the point $(0, -5)$ and parallel to the straight line whose equation is $y = 2x + 3$?

A. $x + 2y + 5 = 0$
•B. $2x - y - 5 = 0$
C. $2x + 3 = -5$
D. $2x - 5y + 3 = 0$
E. $2x + y + 5 = 0$

2. An open cylindrical vessel of capacity $9{,}600\pi$ cc is to be made with the curved surface of sheet metal and a wooden base. If the weight of 1 sq cm of the metal is three times the weight of 1 sq cm of the wood, each being of uniform small thickness, what will be the radius of the vessel (in cms) when its total weight is a minimum?
Ans. 30

3. The derivative with respect to x of $\frac{4}{\sqrt{3x-4}}$ is

A. $12\sqrt{3x-4}$
B. $4/\sqrt{3}$
C. $\frac{-2}{(3x-4)^{3/2}}$
•D. $\frac{-6}{(3x-4)^{3/2}}$
E. $6\sqrt{3x-4}$

4. The value of $\int_0^1 \frac{dx}{x^2 - 5x + 6}$ is:

A. $\frac{1}{2} \log_e 2$
B. $\frac{1}{3}$
•C. $\log_e \frac{4}{3}$
D. $\tan \frac{-11}{4}$
E. $\frac{1}{2}$

x3 – Geo's Master II

Test Item

5. $\int (x-1)^2\,dx$ is equal to

A. $2(x-1)+c$ D. $\frac{1}{3}(x^3-x)+c$

B. $\frac{1}{2}(x-1)^3+c$ E. $\frac{(x-1)^3}{x}+c$

*C. $\frac{1}{3}(x-1)^3+c$

6. Determine k so that the graph of the function $y=3x^3+6x^2+kx+9$ has a point of inflection and a horizontal tangent for the same value of x. *Ans. 4*

7. What is the equation in x and y of the curve with parametric equations

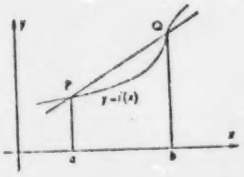
$$x=t+\frac{1}{t},\quad y=t-\frac{1}{t}?$$

A. $x+y=1$ *D. $x^2-y^2=4$

B. $x+y=2$ E. $2x^2-y^2=4$

C. $x^2+y^2=4$

8. The graph of a polynomial function of x is shown in the diagram to the right, the equation of the curve being $y=f(x)$. Which of the following statements is (are) true for that part of the curve for which $a\leq x\leq b$?



I. $f''(c)=0$ for some value c between a and b.

II. $\frac{f(b)-f(a)}{b-a}=f'(c)$ for some value c between a and b.

III. If there is a point of inflexion at Q, $f''(b)$ can have no value but 0.

IV. $\int_a^b f(x)dx<\frac{1}{2}(b-a)\,[f(a)+f)b)]$

A. All four D. I and III

*B. II, III, and IV E. II and IV

C. I and II

9. Given that $3\frac{dy}{dx}=x^3-5$, and $y_1=1$ when $x=2$.

What is the value of y when $x=0$?

A. $-5/3$ B. $-2/3$ C. 1/3 D. 25/9 *E. 31/9

Problems 10 and 11 are based on the figure on the right, which shows a graph of $y=f(x)$, a being less than b. S_1 is the area enclosed by the x-axis, $x=a$, and $y=f(x)$. S_2 is the area enclosed by the x-axis, $x=b$, and $y=f(x)$. S_1 and S_2 are to be considered positive.

Test Item

10. The value of $\int_a^b f(x)dx$ is

A. S_1+S_2 D. $|S_1-S_2|$

B. S_1-S_2 E. $\frac{1}{2}(S_1+S_2)$

*C. S_2-S_1

11. The value of $\int_a^b |f(x)|\,dx$ is

*A. S_1+S_2 D. $|S_1|-|-S_1|$

B. S_1-S_2 E. $\frac{1}{2}(S_1+S_2)$

C. $|S_2-S_1|$

12. The function $f(x)=\frac{x^3-1}{x-1}$ is defined and continuous for all x except $x=1$. What value must be assigned to $f(x)$ for $x=1$ in order that the function be continuous there? *Ans. 2*

13. Find the difference $\vec{b}-\vec{a}$ of the vectors $\vec{a}=(4,2)$ and $\vec{b}=(0,3)$.

A. $(-4,-2)$ B. $(-4,1)$ *C. $(4,-1)$ D. $(4,2)$ E. $(4,5)$

14. In a triangle with area a, the midpoints of the three sides are joined so as to form a new triangle. In the triangle thus constructed, another new triangle is inscribed in the same way. This process is continued indefinitely. What is the sum of all of the areas of this sequence of triangles, including the original one?

A. $\frac{9a}{7}$ *B. $\frac{4a}{3}$ C. $\frac{7a}{5}$ D. $\frac{3a}{2}$ E. $\frac{5a}{3}$

15. The value of $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$ is

A. 0 *B. $\frac{1}{2\sqrt{2}}$ C. $\frac{1}{2}$ D. $\frac{1}{\sqrt{2}}$ E. ∞

TASKS FOR NATIONAL COMMITTEES IN RELATION TO WORKING PAPER I

DETAILS OF MATHEMATICS SYLLABUSES

In addition to providing comments, suggestions and criticisms to Working Paper 1 would you please provide the following information.

1. Please provide details of the various mathematics courses available to students in your country. (Use separate sheet if more convenient.)

Population A	Name of Course	Percentage of cohort following course
Course (a)		
(b)		
(c)		
(d)		
(e)		
(f)		

Population B	Name of Course	Percentage of cohort following course
Course (g)		
(h)		
(i)		
(j)		
(k)		
(l)		

2. Please provide official statements on details of the mathematics syllabuses current in your country for each course available to Populations A and B.

3. Please provide lists or preferably copies of the textbooks or other materials normally used for each course available to Population A and Population B. (Note. The International Co-ordinating Center already has some of this information from Japan and Ireland.)

4. Please provide copies of typical examination papers and standardized tests for the various stages of the courses available to Population A and Population B.

5. Please give details of any experimental mathematical programs currently under way or to be implemented by 1980 and indicate the approximate percentage of students likely to be involved in these programs. (Use separate sheet please.)

6. For each population and for each major course (ie, a course being followed by more than 10% of the appropriate student groups) please complete the attached content lists to indicate the extent to which the proposed content dimensions are included in the course curriculum. (Additional copies will need to be made for each course.)

   Indications are required at two levels:

   (1) in terms of Universality, ie, the topic is covered by All (A), Some (S) or None (O) of the students following the course;

   (2) in terms of the Degree of emphasis placed on the topic in the course curriculum, ie, very important (V), important (I), not important (N).

   Working Paper VI also asks that you add a minimal competencies classification to the list.
   Additional topics not already specified and which countries wish to add should be entered on the content list and classified in terms of Universality and Degree of emphasis. Space has been left in each section for these additions.

7. For each population and for each major course (ie, a course being followed by more than 10% of the appropriate student groups) please complete the attached content/behaviors grids. (Additional copies will need to be made for each course.) A restricted content list in terms of the behavioral objectives has been used. Space again has been left for countries to add any major content areas not included in the content dimension.

   The following information is required for each cell:

   (1) for each area of content the degree of emphasis for the particular behavioral aim should be classified as follows:

Very important	-	V
Important	-	I
Not Important	-	N

(2) for each cell of the grid please make an estimate of the percentage of overall teaching time devoted to the particular content area and the associated behavioral objective.

Please enter the appropriate letter and percentage in each cell of the grids.

8. For each cell in the content/behaviors grids in which a "V" has been entered please supply two test items relating to the content area specified which assess the associated behavioral objective.

9. For each of the following cells selected from the content/behavior grids, please supply an item which is typically representative of content topic and behavioral level appropriate for the most popular course in the country.

Please identify the course alongside the item.

Population A

	Content Topic	Behavior
1.	002.5	Computation
2.	001.6	Application
3.	104.1	Application
4.	212	Analysis
5.	110.3	Analysis
6.	300.3	Comprehension
7.	001.2	Computation
8.	203.2	Computation
9.	401.3	Comprehension
10.	106.1	Comprehension

Population B

	Content Topic	Behavior
1.	5.5.1	Application
2.	1.2	Comprehension
3.	2.4.1	Computation
4.	3.6	Comprehension
5.	3.3.2	Computation
6.	4.3.1	Application
7.	6.1.7	Analysis
8.	7.1	Comprehension
9.	4.6.2	Analysis
10.	5.4.1	Computation

10. Please complete the following table (if precise statistical data are not available please provide estimates) indicating the numbers of students following particular mathematics courses by sex, type of course and grade.

TABLE - Mathematics Courses by Sex, Type and Grade
See Question 1 for key to "a-l".

US Grade	National Equivalent Classification	(a)			(b)			(c)			(d)			(e)			(f)			(g) -			(h)			(i)			(j)			(k)			(l)			Total		
		M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T
K																																						
1																																						
2																																						
3																																						
4																																						
5																																						
6																																						
7																																						
8																																						
9																																						
10																																						
11																																						
12																																						
13																																						
Total																																						

These figures are Official statistics

Only estimates (Delete which does not apply.)

11. For each course specified in Q.1 please indicate the criteria (if any) or other requirements which determine admission to the course.

	Criteria for Admission
Course (a)	
(b)	
(c)	
(d)	
(e)	
(f)	
Course (g)	
(h)	
(i)	
(j)	
(k)	
(l)	

12. For each course specified in Q.1 what is the average time per week (in minutes) devoted to mathematics instruction classified by grade?

COURSE

US Grade	National Equivalent Grade	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)
K													
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													
13													

13. What is the total number of hours of schooling per week provided in each grade in your country (exclude all lunch breaks and other intervals)?

US Grade	National Equivalent Grade	Total Instruction Time	
		Hours	Minutes
K			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			

14. (a) To what extent can students choose to follow an essentially specialised Mathematics program at the Population B stage?

(ie, more than 30% of their total instructional time.)

(b) What are the numbers of students involved in such specialization?

Male _____ Female _____

15. Please complete the following table giving details of the numbers of students following the first-year courses in mathematics in post secondary-school institutions.

Type of Course (please specify)	Duration of Course in Years	Type of Institution (please specify in Qu.17b)															
		(a)			(b)			(c)			(d)			(e)			(f)	
		M	F	T	M	F	T	M	F	T	M	F	T	M	F	T	M	F	T
A																			
B																			
C																			
D																			
E																			
F																			
G																			
H																			
Total																			

16. Please complete the following table indicating the number of students per year successfully completing the full Mathematics courses specified in Q.15.

Type of Course	Number of students successfully completing course per year.		
	Male	Female	Total
A			
B			
C			
D			
E			
F			
G			
H			
Total			

17. Please amplify the name of the courses and institutions specified in Q.15. (Use separate sheet if more convenient.)

(a)

Course	Name	Translation in English	Brief Description of Course
A			
B			
C			
D			
E			
F			
G			
H			

(b)

Institution	Name	English Translation	Brief Description of Type
a			
b			
c			
d			
e			
f			

18. What percentage of all students graduate (ie, equivalent of an English B.A.) with mathematics as one of their subjects? _____%

What percentage of female students graduate (ie, equivalent of an English B.A.) with mathematics as one of their subjects? _____%

What percentage of male students graduate (ie, equivalent of an English B.A.) with mathematics as one of their subjects? _____%

QUESTION 6 (Attachment)

CONTENT LIST CLASSIFIED FOR UNIVERSALITY AND DEGREE OF EMPHASIS.

Country _____

Population A

Course _____

(Code as per Question 1)

Coding:-

Universality (U)

A - All
S - Some
0 - None

Degree of Emphasis (E)

V - Very important
I - Important
N - Not important

Minimal Competencies (M)

M - Important

	Content List	U	E	M	Comments
000	Arithmetic				
001	Natural numbers and whole numbers				
	001.1 Place value				
	001.2 Operations (function, algorithms)				
	001.3 Representation on number line				
	001.4 Properties of operations				
	001.5 Ordering whole numbers				
	001.6 Estimating products, sums, quotients, differences				
002	Common fractions				
	002.1 Fractional parts				
	002.2 Order, equivalent fractions				
	002.3 Representation on a number line				
	002.4 Simplification of fractions				
	002.5 Mixed numerals				
	002.6 Density				
	002.7 Operations (addition, subtraction, multiplication, division)				
	002.8 Approximation to simpler fractions				
	002.9 Estimation				

003 Decimal fractions

003.1 Common fractions with denominators a power of ten

003.2 Place value

003.3 Order of decimal fractions

003.4 Representation on number line

003.5 Representation of rational numbers by decimal fractions

003.6 Estimation and approximation

003.7 Conversion of fractions to decimals

003.8 Operations (addition, subtraction, multiplication, division)

004 Ratio, proportion, percentage

004.1 Equivalence of ratios, proportions

004.2 Conversion: percentage, fraction

005 Number theory

005.1 Divisibility tests

005.2 Primes and composites

005.3 Prime factorization

005.4 Relation between divisors, factors, multiples

005.5 Greatest common divisor (gcd) and least common multiple (lcm)

006 Powers and exponents

006.1 Base, exponent, power

006.2 Powers of ten, scientific notation

007 Other numeration systems

007.1 Place value

007.2 Counting

007.3 Converting: decimal and non-decimal

007.4 Operations in binary system

008 Square roots

008.1 Definition

008.2 Estimations

008.3 Reading table of square roots

008.4 Calculation

009 Dimensional analysis (denominate numbers)

009.1 Addition, subtraction, multiplication and division of denominate numbers

009.2 Units associated with area, volume

009.3 Units associated with speed, pressure, etc

100 Algebra

101 Integers

101.1 Extension of whole numbers to integers, number line

101.2 Order

101.3 Absolute value

101.4 Operations, extension to negative integers

101.5 Properties of operations

102 Rationals

102.1 Positive - negative

102.2 Graphical representation

102.3 Order

102.4 Density

102.5 Equivalence

102.6 Operations

102.7 Properties of operations

103 Integer exponents

103.1 Extension of exponents to integers

103.2 Laws of exponents

4 Formulas and algebraic expressions

104.1 Variables

104.2 Algebraic expressions

104.3 Evaluation of formulas or algebraic expressions

104.4 Derivation of formulas and expressions from given information

Polynomials and rational expressions

105.1 Polynomials

105.2 Simplification

105.3 Operations: +, -, x

105.4 Special forms $(x + y)^2$, x^2-y^2

105.5 Factoring

105.6 Rational expressions

Equations and inequations

106.1 Linear equations

106.2 Quadratic equations

106.3 Rational equations

106.4 Algebraic, (contains roots of variable) equations

106.5 Solution of linear inequations

106.6 Equations and inequations involving absolute value

7 Relations and functions

107.1 Representation of relations: formula, table, graph, sets of ordered pairs

107.2 Single-valued relations - functions

107.3 Describing functions by rules, constant, linear, quadratic functions

Systems of linear equations

108.1 Interpreting information as linear equations

108.2 Graphical and algebraic solution: elimination, substitution, Cramer's Rule

108.3 Inconsistent systems

108.4 Dependent systems

109 Finite systems

109.1 Modular arithmetic tables

109.2 Comparative structure (commutative, associative, etc.)

110 Finite sets

110.1 Specifying a set

110.2 Operations and relations on sets: $\in, \subset, \cup, \cap$, complementation, differences

110.3 Representation of sets

110.4 Properties of operations

111 Flowcharts and programming

111.1 Make flowcharts

111.2 Interpret flowcharts

111.3 Making algorithms

111.4 Programming

112 Real numbers

112.1 Decimal representation

112.2 Operations with irrational numbers

200 Geometry

201 Classification of plane figures: number of edges, parallel sides

202 Properties of plane figures

203 Congruence of plane figures

203.1 Classification by congruence

203.2 Properties of congruent figures

204 Similarity of plane figures

204.1 Classification

204.2 Recognition

204.3 Scale Drawings

205 Geometric constructions

205.1 Ruler and compass

205.2 Mira construction (Mirror cards)

206 Pythagorean triangles

206.1 Using Pythagoras' theorem

206.2 Pythagorean triples

206.3 Irrational numbers as lengths of hypotenuse

207 Coordinates

207.1 Location of points

207.2 Graphing relations

208 Simple deductions

209 Informal transformations in geometry

210 Relationships between lines and planes in space

211 Solids (symmetry properties)

212 Spatial visualization and representation

213 Orientation (spatial)

213.1 In plane

213.2 In space

214 Decomposition of figures

300 Probability and Statistics

301 Data collection

302 Organization of data

303 Representation of data

304 Interpretation of data

304.1 Central tendency

304.2 Dispersion

304.3 Percentiles

305 Combinatorics

306 Outcomes, sample spaces and events

307 Counting of sets, $P(A \cap B)$, $P(A \cup B)$, independent events

308 Mutually exclusive events

309 Complementary events

400 Measurement

401 Standard units of measure

401.1 System of units

401.2 Relation between units

401.3 Appropriate units

402 Estimation

403 Approximation: precision, accuracy

404 Determination of measures: areas, volumes, etc

QUESTION 6 (Attachment)

CONTENT LIST CLASSIFIED FOR UNIVERSALITY AND DEGREE OF EMPHASIS.

Country _____

Population B

Course _____

(Code as per Question 1)

Coding:-

Universality (U)

A - All
S - Some
O - None

Degree of Emphasis (E)

V - Very important
I - Important
N - Not important

Minimal Competencies (M)

M - Important

Content List	U	E	M	Comments
Sets, Relations and Functions				
.1 Set notation				
.2 Set operations (e.g. union, inclusion)				
.3 Relations				
1.3.1 Definitions and representations				
1.3.2 Properties (symmetric, etc.)				
1.3.3 Operations on relations (inverse and composition)				
.4 Functions				
1.4.1 Definitions and representations, connections with relations				
1.4.2 Properties (1-1, onto, etc.)				
1.4.3 Operations on functions (inverse, composition)				
.5 Infinite sets, cardinality and cardinal algebra (rationals and reals)				
Number Systems				
.1 Common laws for number systems (e.g. $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$)				
.2 Natural numbers				
2.2.1 Induction, recursive definitions				
2.2.2 Elements of number theory				

2.3 Decimals

2.3.1 Finite decimals

2.3.2 Approximation by finite decimals

2.3.3 Infinite decimals, periodicity

2. Real numbers

2.4.1 Arithmetic of radicals

2.4.2 Powers, with rational exponents

2.4.3 Least upper bound and greatest lower bound

.5 Complex numbers

2.5.1 Addition and subtraction

2.5.2 Geometric representation

2.5.3 Absolute value and argument

2.5.4 Polar representation

2.5.5 Multiplication and division

2.5.6 De Moivre's formula

Algebra

. Polynomials (over R)

3.1.1 Addition and multiplication

3.1.2 Factorization

3.1.3 Evaluation, roots or zeros

3.1.4 Degree - number of roots (or zeros)

3.1.5 Quadratic equations

3.1.6 Polynomial equations of degree > 2

.2 Quotients of polynomials

3.2.1 Addition, subtraction, multiplication and division.

3.2.2 Partial fractions

(eg $\frac{2x - 1}{x^2-3x + 2} = \frac{3}{x-2} - \frac{1}{x-1}$)

3.3 Roots and radicals

3.3.1 Simplification (e.g. $\sqrt{a^2b} = a\sqrt{b}$)

3.3.2 Rationalizing denominators

(e.g. $\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{a-b}$)

3.4 Equations and inequalities

3.4.1 Linear equations

3.4.2 Quadratic equations

3.4.3 Polynomial equations of degree > 2

3.4.4 Linear inequalities

3.4.5 Quadratic inequalities

3.4.6 Inequalities of quotients (e.g. $\frac{x}{x-1} < 0$)

3.4.7 Inequalities and equations involving absolute value
(e.g. $|3x + 2| + |2x - 1| < 4$)

3.5 Systems of equations and inequalities

3.6 Matrices and determinants

3.7 Groups, rings and fields

3.7.1 Definitions and examples

3.7.2 Properties

Geometry

4.1 Euclidean (synthetic) geometry

4.1.1 Informal treatment

4.1.2 Axiomatic treatment - Euclid's axioms

4.1.3 Axiomatic treatment - transformations

4.2 Affine and projective geometry in the plane

4.3 Analytic (coordinate) geometry in the plane

4.3.1 Lines: slope, equations of, parallel, perpendicular, families

4.3.2 Distance: between points, from point to line

4.3.3 Conics: definitions, equations, properties

.4 Three-dimensional coordinate geometry

.5 Vector methods

.6 Trigonometry

4.6.1 Right angled triangles

4.6.2 General triangles

4.6.3 Trigonometric identities (formulas)

.7 Finite geometries (e.g.projective or affine geometry with a finite set of points)

.8 Elements of topology (e.g.Euler's formula for polyhedra)

Analysis

.1 Elementary functions

5.1.1 Polynomial functions

5.1.2 Rational functions

5.1.3 Algebraic functions (e.g.$f(x) = \sqrt{x}$)

5.1.4 Circular functions

5.1.5 Inverse circular functions

5.1.6 Exponential functions

5.1.7 Logarithmic functions

.2 Properties of functions

5.2.1 Domain and range

5.2.2 Composition

5.2.3 Inverse

.3 Limits and continuity

5.3.1 Definition and examples

5.3.2 Theorems (e.g.limits of sums of functions, of products, of quotients)

5.3.3 Asymptotes of functions

5.3.4 Limits of sequences and series of constants

.4 Differentiation

5.4.1 Definition and examples

5.4.2 Of elementary functions

5.4.3 Of a sum

5.4.4 Of a product

5.4.5 Of a quotient

5.4.6 Of a composite (chain rule)

5.4.7 Of the inverse of a function

5.4.8 Logarithmic differentiation

.5 Applications of the derivative

5.5.1 For finding maxima and minima

5.5.2 To graphing of functions

5.5.3 Mean value theorem

5.5.4 Extended Mean Value Theorem (Taylor's formula)

5.5.5 Differentials (approximation)

.6 Integration

5.6.1 Numerical integration - approximating the area under a curve

5.6.2 Definite integral

5.6.3 Functions defined by integrals (e.g. $\text{Log } x = \int_1^{x_1} \frac{1}{t}\, dt$)

5.6.4 Antiderivative

5.6.5 Fundamental theorem of calculus

.7 Techniques of integration

5.7.1 Integration of a linear combination of functions

5.7.2 Integration by substitution

5.7.3 Integration by parts

5.7.4 Integration by partial fractions

.8 Applications of integration

5.8.1 Areas

5.8.2 Volumes of rotation

5.8.3 Other volumes

5.8.4 Arc length

.9 Differential equations

5.9.1 Definition and examples

5.9.2 First order linear differential equations

5.9.3 Second order linear differential equations with constant coefficients
5.9.4 Separable equations
5.9.5 Homogeneous equations
10 Sequences and series of functions
5.10.1 Power series
5.10.2 Maclaurin and Taylor series

Probability and Statistics
.1 Probability
6.1.1 Sample space (set of possible outcomes)
6.1.2 Probability of an event
6.1.3 Independent events
6.1.4 Conditional probability
6.1.5 Probability theorems (e.g. Bayes' theorems)
6.1.6 Bernoulli trials
6.1.7 Expectation
6.1.8 Markov chains
6.1.9 Random variables
6.1.10 Monte Carlo methods
.2 Statistics
6.2.1 Data collection and organization
6.2.2 Data representation
6.2.3 Interpretation
. Central tendency
. Dispersion
. Percentiles
. Measures of relationship

.3 Distributions

6.3.1 Normal distributions

6.3.2 Binomial distributions

6.3.3 Central limit theorem

.4 Statistical inference

6.4.1 Point estimation

6.4.2 Interval estimation

6.4.3 Sampling distributions

6.4.4 Standard error

6.4.5 Hypothesis testing

- Type I error
- Type II error

6.4.6 Tests of significance

- Z test
- t test
- X^2 test

.5 Bivariate statistics

6.5.1 Regression

6.5.2 Correlation

Finite mathematics

.1 Combinatorics

.2 Game theory

. Graphs and networks

3 omputer Science
3. Flowcharts

3. Computer languages and programming

3.3 Algorithms

3. Data processing

9 Logic

9.1 Propositional logic

9.1.1 Boolean algebra

9.1.2 Switching circuits

9.2 Predicate logic

9.2.1 Quantifiers (eg. "there exists", "for all")

9. Rules of inference

9.4 Axiomatics; deductive systems

QUESTION 7 (Attachment)

RESTRICTED CONTENT LIST IN TERMS OF BEHAVIORAL OBJECTIVES CLASSIFIED FOR

1: Emphasis of behavioral aims.

2: Percentage of teaching time devoted to each cell.

Please enter the appropriate letter and percentage in each cell of the grid.

Coding:-

Degree of emphasis

V - Very important

I - Important

N - Not important

Country _____

Population A

Course _____

(Code as per question 1)

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
000 Arithmetic 001 Natural numbers and whole numbers				
002 Common fractions				
003 Decimal fractions				
004 Ratio, proportion, percent				
005 Number theory				
006 Powers and exponents				
007 Other numeration systems				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
008 Square roots				
009 Dimensional analysis (denominate numbers)				
010 Algebra				
101 Integers				
102 Rationals				
103 Integer exponents				
104 Formulas and algebraic expressions				
105 Polynomials and rational expressions				
106 Equations and inequations				
107 Relations and functions				
108 Systems of linear equations				
109 Finite systems				
110 Finite sets				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
111 Flowcharts and programming				
112 Real numbers				
200 Geometry				
201 Classification of plane figures				
202 Properties of plane figures				
203 Congruence of plane figures				
204 Similarity of plane figures				
205 Geometric constructions				
206 Pythagorean triangles				
207 Coordinates				
208 Simple deductions				
209 Informal ansformations in geometry				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
[illegible] Relationships between lines and planes in space				
211 Solids (symmetry properties)				
[illegible] Spatial visualization and representation				
213 Orientation (spatial)				
214 Decomposition of figures				
[illegible]00 Probability and Statistics				
[illegible]30 Data collection				
302 Organisation of data				
[illegible]03 Representation of data				
[illegible]04 Interpretation of data				
[illegible]05 Combinatorics				
[illegible]0 Outcomes, sample spaces and events				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
307 Counting of sets, P(A∩B), P(A∪B), independent events				
308 Mutually exclusive events				
[illegible] Complementary events				
400 Measurement				
[illegible] Standard units of measure				
402 Estimation				
[illegible] Approximation				
[illegible] Determination of measures				

QUESTION 7 (Attachment)

RESTRICTED CONTENT LIST IN TERMS OF BEHAVIORAL OBJECTIVES CLASSIFIED FOR

1. Emphasis of behavioral aims.
2. Percentage of teaching time devoted to each cell.

Coding:-

Degree of emphasis

V - Very important

I - Important

N - Not important

Please enter the appropriate letter and percentage in each cell of the grid.

Country _____

Population B

Course _____

(Code as per question 1)

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
Sets, Relations and Functions 1.1 Set notation				
1.2 Set operations e.g. union, inclusion)				
1.3 Relations				
1.4 Functions				
1.5 Infinite sets, cardinality and cardinal algebra				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
2 Number Systems				
2.1 Common laws for number systems				
2.2 Natural numbers				
2.3 Decimals				
2.4 Real numbers				
2.5 Complex numbers				
3 Algebra				
3.1 Polynomials (over R)				
3.2 Quotients of polynomials				
3.3 Roots and radicals				
3.4 Equations and inequalities				
3.5 Systems of equations and inequalities				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
3.6 Matrices and determinants				
3.7 Groups, rings and fields				
4 Geometry				
.1 Euclidean (synthetic) geometry				
.3 Affine and projective geometry in the plane				
.5 Analytic (coordinate) geometry in the plane				
.4 Three-dimensional coordinate geometry				
.5 Vector methods				
.6 Trigonometry				
.7 Finite geometries				
.8 Elements of topology				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
5 Analysis				
.1 Elementary functions				
5.2 Properties of functions				
.3 Limits and continuity				
5.4 Differentiation				
5.5 Applications of the derivative				
5.6 Integration				
5.7 Techniques of integration				
5.8 Applications of integration				
.9 Differential equations				
.10 Sequences and series of functions				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
Probability and statistics				
.1 Probability				
5.2 Statistics				
5. Distributions				
.4 Statistical inference				
.5 Bivariate statistics				
Finite mathematics				
.1 Combinatorics				
.2 Game theory				
.3 Graphs and networks				

CONTENT LIST	COMPUTATION BEHAVIOR	COMPREHENSION BEHAVIOR	APPLICATION BEHAVIOR	ANALYSIS BEHAVIOR
8 - Computer Science				
8.1 Flowcharts				
8. Computer languages and programming				
8.3 Algorithms				
Data processing				
9 Logic				
.1 Propositional logic				
.2 Predicate logic				
.3 Rules of inference				
.4 Axiomatics; deductive systems				